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Fixed point free involutions on Riemann surfaces. (English) Zbl 1157.30031

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Let S be an orientable surface of even genus with a Riemannian metric d and with a fixed point free, orientation reversing involution τ. Then it is conjectured that there exists a point $p \in S$ satisfying

$$\frac{d(p,\tau(p))^2}{\text{area}(S)} \leq \frac{\pi}{4}.$$

This conjecture originated from the filling area conjecture by *M. Gromov* [J. Differ. Geom. 18, 1–147 (1983; Zbl 0515.53037)]. In the case that S is hyperelliptic, it was positively solved by *V. Bangert, C. Croke, S. Ivanov*, and *M. Katz* [Geom. Funct. Anal. 15, No. 3, 577–597 (2005; Zbl 1082.53033)]. The situation is different when S has odd genus. One of the main results in this paper is that for any odd $g \geq 3$ and positive constant k, there exists a hyperbolic Riemann surface S of genus g with an orientation reversing involution τ such that $d(p,\tau(p)) > k$ holds for all $p \in S$. This result is true in the case that τ is an orientation preserving involution. The other main result concerns the sharp bound for hyperbolic metrics in genus 2 surfaces, that is, for a Riemann surface S of genus 2 with a hyperbolic metric and with an involution τ, there exists a point $p \in S$ satisfying $d(p,\tau(p)) \leq \operatorname{arccosh}\frac{5+\sqrt{17}}{2}$. It is mentioned that the surface which attains the sharp bound is not in the conformal class of the Bolza curve.

Reviewer: Gou Nakamura (Toyota)

MSC:

30F45 Conformal metrics (hyperbolic, Poincaré, distance functions)
53C23 Global geometric and topological methods (à la Gromov); differential geometric analysis on metric spaces
30F50 Klein surfaces

Keywords:

hyperbolic Riemann surface; hyperelliptic Riemann surface; involution; hyperbolic metric; systole

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