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The one-phase Hele-Shaw problem with singularities. (English) Zbl 1155.35496

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Summary: In this article we analyze viscosity solutions of the one phase Hele-Shaw problem in the plane and the corresponding free boundaries near a singularity. We find, up to order of magnitude, the speed at which the free boundary moves starting from a wedge, cusp, or finger-type singularity. Maximum principle-type arguments play a key role in the analysis.

MSC:

35R35 Free boundary problems for PDEs
35Q35 PDEs in connection with fluid mechanics
76D27 Other free boundary flows; Hele-Shaw flows

Keywords:

Hele-Shaw problem; viscosity; solutions; free boundary; wedge; cusp; finger-type singularity

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