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On the correlation structure of a Lévy-driven queue. (English) Zbl 1154.60348

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Summary: We consider a single-server queue with Lévy input and, in particular, its workload process $(Q_t)_{t\geq 0}$, with a focus on the correlation structure. With the correlation function defined as $r(t) := \mathrm{cov}(Q_0, Q_t)/var(Q_0)$ (assuming that the workload process is in stationarity at time 0), we first determine its transform $\int_0^\infty r(t)\mathrm{e}^{-\vartheta t}\mathrm{d}t$. This expression allows us to prove that $r(\cdot)$ is positive, decreasing, and convex, relying on the machinery of completely monotone functions. We also show that $r(\cdot)$ can be represented as the complementary distribution function of a specific random variable. These results are used to compute the asymptotics of $r(t)$, for large t, for the cases of light-tailed and heavy-tailed Lévy inputs.

MSC:

60K25 Queueing theory (aspects of probability theory)
60G51 Processes with independent increments; Lévy processes
90B05 Inventory, storage, reservoirs

Keywords:

Lévy process; reflection; correlation; complete monotonicity

Full Text: DOI

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