zbMATH Open
THE FIRST RESOURCE FOR MATHEMATICS

Papageorgiou, Nikolaos S.; Rocha, Eugénio M.

A multiplicity theorem for a variable exponent Dirichlet problem. (English) Zbl 1154.35041

Glasg. Math. J. 50, No. 2, 335-349 (2008).

If $N \in \mathbb{N}$ and $\Omega \subseteq \mathbb{R}^N$ is a bounded domain with smooth boundary, consider the problem

$$\begin{aligned} -\operatorname{div}(|\nabla u|^{p(x)-2}\nabla u) = m(x)|u|^{q-2}u + f(x,u), &\quad \text{in } \Omega, \\ u = 0, &\quad \text{on } \partial\Omega, \end{aligned} \tag{1}$$

where $p \in C^1(\overline{\Omega})$, $\min_{\overline{\Omega}} p > q > 1$, $m \in L^\infty(\Omega)\backslash\{0\}$, $m \geq 0$, and f is a Carathéodory function with subcritical growth in a suitable sense, with respect to the variable exponent $p(x)$. The authors prove the existence of three classical solutions to (1) that are ordered, and such that one solution is negative and one positive. The method consists in a combination of the sub-supersolution technique and the Mountain Pass Theorem applied to a suitably truncated functional. One should compare this result with the recent article by *J. Yao* [Nonlinear Anal., Theory Methods Appl. 68, No. 5 (A), 1271–1283 (2008; Zbl 1158.35046)], where a similar result is achieved under Neumann boundary conditions and with a different set of hypotheses, including the Ambrosetti-Rabinowitz condition.

Reviewer: Nils Ackermann (México)

MSC:

35J60 Nonlinear elliptic equations
35J70 Degenerate elliptic equations
35J65 Nonlinear boundary value problems for linear elliptic equations
35J20 Variational methods for second-order elliptic equations
35B50 Maximum principles in context of PDEs

Keywords:

varying exponent; Dirichlet boundary values; subcritical nonlinear equation; sub- and supersolution

Full Text: DOI

References:

[1] DOI: 10.1016/S0362-546X(97)00628-7 · Zbl 0927.46022 · doi:10.1016/S0362-546X(97)00628-7
[2] DOI: 10.1112/S0024610705006952 · Zbl 1161.35405 · doi:10.1112/S0024610705006952
[3] DOI: 10.1016/S0022-1236(02)00103-9 · Zbl 1091.35028 · doi:10.1016/S0022-1236(02)00103-9
[4] DOI: 10.1016/j.jde.2007.01.008 · Zbl 1143.35040 · doi:10.1016/j.jde.2007.01.008
[5] DOI: 10.1016/j.jde.2004.03.019 · Zbl 1079.35035 · doi:10.1016/j.jde.2004.03.019
[6] DOI: 10.1016/j.jmaa.2006.07.093 · Zbl 1206.35103 · doi:10.1016/j.jmaa.2006.07.093
[7] DOI: 10.1016/j.jfa.2006.11.015 · Zbl 1231.35085 · doi:10.1016/j.jfa.2006.11.015
[8] DOI: 10.1017/S0017089506003144 · Zbl 1387.35289 · doi:10.1017/S0017089506003144
[9] DOI: 10.1155/S1085337502207010 · Zbl 1106.35308 · doi:10.1155/S1085337502207010
[10] Mawhin, Critical point theory and Hamilton systems (1989)· doi:10.1007/978-1-4757-2061-7
[11] DOI: 10.1016/j.jde.2003.08.001 · Zbl 1087.35034 · doi:10.1016/j.jde.2003.08.001
[12] DOI: 10.1112/S0024609304004023 · Zbl 1122.35033 · doi:10.1112/S0024609304004023
[13] DOI: 10.1016/j.jmaa.2005.04.034 · Zbl 1148.35321 · doi:10.1016/j.jmaa.2005.04.034
[14] DOI: 10.1007/s00205-002-0208-7 · Zbl 1038.76058 · doi:10.1007/s00205-002-0208-7
[15] Kov??ik, Czechosloval Math. J. 41 pp 592– (1991)
[16] DOI: 10.1016/S0022-247X(03)00165-3 · Zbl 1146.35358 · doi:10.1016/S0022-247X(03)00165-3
[17] DOI: 10.1016/S0362-546X(02)00150-5 · Zbl 1146.35353 · doi:10.1016/S0362-546X(02)00150-5