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Multiple solutions of semilinear elliptic equations in exterior domains. (English)

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Let Ω denote a smooth exterior domain in \mathbb{R}^N , $q : \Omega \rightarrow \mathbb{R}$ be positive and continuous, and $p \in (2, 2^*)$, where $2^* = \infty$ if $N = 1, 2$ and $2^* = 2N/(N - 2)$ if $N \geq 3$. The author considers solutions to the problem

$$\begin{cases} -\Delta u + u = q(x)|u|^{p-2}u, & \text{in } \Omega, \\ u \in H_0^1(\Omega). \end{cases} \quad (1)$$

It is assumed that $q_\infty := \lim_{|x| \rightarrow \infty} q(x) > 0$ exists and that q is not constant. The following results are proved:

Theorem 1: Suppose that there are $C > 0$ and $\delta \in (0, 2)$ such that $q(x) \geq q_\infty + Ce^{-\delta|x|}$. Suppose moreover that q is bounded. Then (1) has two positive solutions.

Theorem 2: Suppose that there are $C > 0$ and $\delta \in (0, 1)$ such that $q(x) \geq q_\infty + Ce^{-\delta|x|}$. Then (1) has two solutions, one positive and one sign changing.

One should compare these theorems with the complementary result in *M. Clapp* and *T. Weth* [*Commun. Partial Differ. Equations* 29, No. 9–10, 1533–1554 (2004; [Zbl 1140.35401](#))].

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MSC:

- [35J60](#) Nonlinear elliptic equations
- [35J20](#) Variational methods for second-order elliptic equations
- [47J30](#) Variational methods involving nonlinear operators

Cited in 4 Documents

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[exterior domain](#); [superlinear subcritical problem](#); [positive solution](#); [sign changing solution](#)

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