Bertola, M.; Mo, M. Y.

Commuting difference operators, spinor bundles and the asymptotics of orthogonal polynomials with respect to varying complex weights. (English) Zbl 1153.39027

Adv. Math. 220, No. 1, 154-218 (2009).

We quote from the abstract in order to describe the three parts of the paper:

(1) "In the first part we apply the theory of commuting pairs of (pseudo) difference operators to the (formal) asymptotics of orthogonal polynomials: using purely geometrical arguments we show heuristically that the asymptotics, for large degrees, of orthogonal polynomial with respect to varying weights is intimately related to certain spinor bundles on a hyperelliptic algebraic curve reproducing formulae appearing in the works of *P. A. Deift* et al. [Ann. Math. (2) 146, No. 1, 149–235 (1997; Zbl 0936.47028)] on the subject.

(2) In the second part we show that given an arbitrary nodal hyperelliptic curve satisfying certain conditions of admissibility we can reconstruct a sequence of polynomials orthogonal with respect to semiclassical complex varying weights supported on several curves in the complex plane. The strong asymptotics of these polynomials will be shown to be given by the spinors introduced in the first part using a Riemann-Hilbert analysis.

(3) In the third part we use Strebel theory [*K. Strebel*, Quadratic differentials (1984; Zbl 0547.30001)] of quadratic differentials and the procedure of welding to reconstruct arbitrary admissible hyperelliptic curves. As a result we can obtain orthogonal polynomials whose zeroes may become dense on a collection of Jordan arcs forming an arbitrary forest of trivalent loop-free trees."

Reviewer: Christian Pötzsche (München)

MSC:

39A70 Difference operators
47B39 Linear difference operators

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Difference operator; Spinor Bundles; Asymptotics of orthogonal polynomials

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