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Asymptotic stability of solitons of the gKdV equations with general nonlinearity. (English)

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Summary: We consider the generalized Korteweg-de Vries equation (gKdV)

$$\partial_t u + \partial_x(\partial_x^2 u + f(u)) = 0, \quad (t,x) \in [0,T) \times \mathbb{R},$$

with general C^3 nonlinearity f. Under an explicit condition on f and $c > 0$, there exists a solution in the energy space H^1 of the type $u(t,x) = Q_c(x - x_0 - ct)$, called soliton. In this paper, under general assumptions on f and Q_c, we prove that the family of solitons around Q_c is asymptotically stable in some local sense in H^1, i.e. if $u(t)$ is close to Q_c (for all $t \geq 0$), then $u(t)$ locally converges in the energy space to some Q_{c+} as $t \to +\infty$. Note, in particular, that we do not assume the stability of Q_c. This result is based on a rigidity property of the gKdV equation around Q_c in the energy space, whose proof relies on the introduction of a dual problem. These results extend the main results in [*Y. Martel*, SIAM J. Math. Anal. 38, No. 3, 759–781 (2006; Zbl 1126.35055); *Y. Martel* and *F. Merle*, J. Math. Pures Appl. (9) 79, No. 4, 339–425 (2000; Zbl 0963.37058), Arch. Ration. Mech. Anal. 157, No. 3, 219–254 (2001; Zbl 0981.35073), Nonlinearity 18, No. 1, 55–80 (2005; Zbl 1064.35171)], devoted to a pure power case.

MSC:

35Q53 KdV equations (Korteweg-de Vries equations)
35Q51 Soliton equations
35B40 Asymptotic behavior of solutions to PDEs

Keywords:

generalized Korteweg-de Vries equation; solitons; asymptotic stability; Liouville property

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