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The method of fundamental solutions for eigenproblems in domains with and without interior holes. (English) [Zbl 1149.35339](#)

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Summary: The main purpose of the present paper is to provide a general method of fundamental solution (MFS) formulation for two- and three-dimensional eigenproblems without spurious eigenvalues. The spurious eigenvalues are avoided by utilizing the mixed potential method. Illustrated problems in the annular and concentric domains are studied analytically and numerically to demonstrate the issue of spurious eigenvalues by the discrete and continuous versions of the MFS with and without the mixed potential method. The proposed numerical method is then verified with the exact solutions of the benchmark problems in circular and spherical domains with and without holes. Further studies are performed in a three-dimensional peanut shaped domain. In the spirit of the MFS, this scheme is free from meshes, singularities and numerical integrations.

MSC:

- 35J05** Laplace operator, Helmholtz equation (reduced wave equation), Poisson equation
- 35P99** Spectral theory and eigenvalue problems for partial differential equations
- 65N25** Numerical methods for eigenvalue problems for boundary value problems involving PDEs

Cited in **24** Documents

Keywords:

method of fundamental solutions; eigenproblems; spurious eigenvalues; mixed potential method

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