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Determining a magnetic Schrödinger operator from partial Cauchy data. (English)

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Suppose that $n \geq 3$ and $\Omega \subseteq \mathbb{R}^n$ is a bounded domain with smooth boundary. For a real magnetic potential $A \in C^2(\overline{\Omega}, \mathbb{R}^n)$ and a bounded electric potential $q \in L^\infty(\Omega)$ consider the magnetic Schrödinger operator

$$\mathcal{L}_{A,q}(x, D) := \sum_{j=1}^{n} (-\mathrm{i}\partial_j + A_j(x))^2 + q(x).$$

It is assumed that 0 is not an eigenvalue of $\mathcal{L}_{A,q} \colon H^2(\Omega) \cap H_0^1(\Omega) \to L^2(\Omega)$. If ν is the exterior unit normal to $\partial\Omega$ then let us define the Dirichlet to Neumann map (DN map)

$$\mathcal{N}_{A,q} \colon H^{1/2}(\partial\Omega) \to H^{-1/2}(\partial\Omega)$$

by

$$\mathcal{N}_{A,q}(f) := ((\partial_\nu + \mathrm{i}A \cdot \nu)\mathcal{L}_{A,q}^{-1} f)|_{\partial\Omega},$$

where $\mathcal{L}_{A,q}^{-1} \colon H^{1/2}(\partial\Omega) \to H^1(\Omega)$ is the Dirichlet inverse of $\mathcal{L}_{A,q}$. If $x_0 \in \mathbb{R}^n \setminus \overline{\mathrm{conv}(\Omega)}$ then

$$F(x_0) := \{x \in \partial\Omega \mid (x - x_0) \cdot \nu(x) \leq 0\}$$

is the front side of $\partial\Omega$ with respect to x_0.

The main result is the following: Suppose that Ω is simply connected, $A_1, A_2 \in C^2(\overline{\Omega}, \mathbb{R}^n)$ are two real magnetic potentials, $q_1, q_2 \in L^\infty(\Omega)$ are two electric potentials, and $\mathcal{L}_{A_k,q_k}$ is invertible in the sense mentioned above, for $k = 1, 2$. If the images of $\mathcal{N}_{A_1,q_1}$ and $\mathcal{N}_{A_2,q_2}$ coincide when restricted to a neighborhood of $F(x_0)$ for some x_0 as above, then A_1 and A_2 differ only by a gradient and $q_1 = q_2$.

In other words, if $n = 3$ the magnetic field and electric potential can be recovered from the values of the images of the DN map on one side of the boundary of the domain. In particular, if Ω is strongly starshaped with respect to a point $x_0 \in \partial\Omega$ then the images of the DN map, restricted to a neighborhood of x_0, determine the magnetic field and the electric potential. This work complements the results in [Ann. Math. (2) 165, No. 2, 567–591 (2007; Zbl 1127.35079)].

Reviewer: Nils Ackermann (México)

MSC:

35R30 Inverse problems for PDEs
35J10 Schrödinger operator, Schrödinger equation
78A46 Inverse problems (including inverse scattering) in optics and electromagnetic theory
78A05 Geometric optics
92C55 Biomedical imaging and signal processing

Keywords:

inverse problem; Dirichlet to Neumann map; magnetic Schrödinger operator; complex geometrical optics solution

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