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Stability of patterns with arbitrary period for a Ginzburg-Landau equation with a mean field. (English) Zbl 1146.35012

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The authors consider the following real elliptic Ginzburg-Landau system with mean field $\langle B\rangle = 0$, on the interval $I := (0, L/2)$:

$$\begin{cases} -A_{xx} = A - A^3 - AB, & t > 0,\ x \in I, \\ -B_{xx} = \mu(A^2)_{xx}, & t > 0,\ x \in I, \\ A_x(0) = A_x(L/2) = 0, \\ B_x(0) = B_x(L/2) = 0. \end{cases} \tag{1}$$

It is assumed that $\mu > 1$ and $L > 0$. Solutions to (1) are in one-to-one correspondence with L-periodic solutions of the equation posed on the real line. Of interest is the existence and linear parabolic stability of solutions to (1) in dependence on the parameter μ. It is shown that any solution of (1) for which A or A_x changes sign is linearly unstable. Therefore the article focuses on positive decreasing solutions.

The main result follows: There are constants $\mu_1 > \mu_2 > 1$ that only depend on L, and that are given explicitly in terms of Jacobi Elliptic Integrals, with the following properties: (a) If $\mu > \mu_1$ then all positive decreasing solutions are constant. (b) If $\mu = \mu_1$ then there is exactly one positive decreasing solution, and it is linearly neutrally stable. (c) If $\mu_1 > \mu > \mu_2$ then there are exactly two positive decreasing solutions. That with smaller value of $A(0)$ is linearly stable, while the other one is linearly unstable. (d) If $\mu_2 \geq \mu > 1$ then there is exactly one positive decreasing solution, and it is linearly unstable. These results give a characterization of the positive decreasing solutions of (1) and a complete bifurcation diagram for this subset of solutions.

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MSC:

35B35	Stability in context of PDEs
35J55	Systems of elliptic equations, boundary value problems (MSC2000)
35B10	Periodic solutions to PDEs
35K57	Reaction-diffusion equations
35K50	Systems of parabolic equations, boundary value problems (MSC2000)
35B32	Bifurcations in context of PDEs

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