

Chiang, Yik-Man; Ruijsenaars, Simon N. M.

On the Nevanlinna order of meromorphic solutions to linear analytic difference equations.

(English) [Zbl 1145.39300](#)

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Summary: For various classes of linear ordinary analytic difference equations with meromorphic coefficients, we study Nevanlinna order properties of suitable meromorphic solutions. For a large class of first-order equations with coefficient of order $\rho \in [0, \infty)$, we explicitly construct meromorphic solutions of order $\leq \rho + 1$. For higher-order equations with coefficients of order $\rho \in [0, \infty)$, we show that meromorphic solutions with increase of order $\leq \rho + 1$ in a certain strip have order $\leq \rho + 1$. The assumptions made in the latter setting may seem quite restrictive, but they are satisfied for several classes of second-order difference equations that have been studied in recent years. The latter include Harper-type equations, "reflectionless" equations, Askey-Wilson-type equations, and equations of relativistic Calogero-Moser type.

MSC:

[39A10](#) Additive difference equations

[30D05](#) Functional equations in the complex plane, iteration and composition of analytic functions of one complex variable

[30D35](#) Value distribution of meromorphic functions of one complex variable, Nevanlinna theory

[34M05](#) Entire and meromorphic solutions to ordinary differential equations in the complex domain

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