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A discontinuous Galerkin scheme based on a space-time expansion. I: Inviscid compressible flow in one space dimension. (English) Zbl 1143.76047

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Summary: We propose an explicit discontinuous Galerkin scheme for conservation laws which is of arbitrary order of accuracy in space and time. The basic idea is to use a Taylor expansion in space and time to define a space-time polynomial in each space-time element. The space derivatives are given by the approximate solution at the old time level, the time derivatives and the mixed space-time derivatives are computed from these space derivatives using the so-called Cauchy-Kovalevskaya procedure. The space-time volume integral is approximated by Gauss quadrature with values at the space-time Gaussian points obtained from the Taylor expansion. The flux in the surface integral is approximated by a numerical flux with arguments given by the Taylor expansions from the left and from the right-hand side of the element interface. The locality of the presented method together with the space-time expansion gives the attractive feature that the time steps may be different in each grid cell. Hence, we drop the common global time levels and propose that every grid zone runs with its own time step which is determined by the local stability restriction. In spite of the local time steps the scheme is locally conservative, fully explicit, and arbitrary order accurate in space and time for transient calculations. Numerical results are shown for the one-dimensional Euler equations with orders of accuracy one up to six in space and time.

MSC:

76M25 Other numerical methods (fluid mechanics) (MSC2010)
76N15 Gas dynamics (general theory)

Keywords:

Discontinuous Galerkin; local time-stepping; arbitrary order; space-time expansion; Cauchy-Kovalevskaya-procedure; generalized Riemann problems

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