

**Hilout, Saïd; Boutat, Mohammed; Grilhé, Jean**

**Plastic deformation instabilities: Lambert solutions of Mecking-Lücke equation with delay.**  
(English) [Zbl 1141.74016](#)

Math. Probl. Eng. 2007, Article ID 45951, 13 p. (2007).

Summary: We study the instabilities during plastic deformation at constant cross-head velocity. The deformation is supposed to be controlled by the emission of dislocation loops. Under some hypothesis analogous to the Mecking-Lücke relation, we derive a linear delay differential-difference equation. The “retarded” time term appears as the phase shift between the time of loop nucleation and the time at which the mean strain is recorded. We show the existence of the solution of strain equation. We give an analytic approach of solution using Lambert functions. The stability is also investigated close to the stable solution using a linearization of the number of nucleated loops functions.

**MSC:**

[74C05](#) Small-strain, rate-independent theories of plasticity (including rigid-plastic and elasto-plastic materials)

Cited in **2** Documents

[74H55](#) Stability of dynamical problems in solid mechanics

[39A11](#) Stability of difference equations (MSC2000)

**Keywords:**

[dislocation loops](#); [differential-difference equation](#); [linearization](#)

**Software:**

[LambertW](#)

**Full Text:** [DOI](#) [EuDML](#)

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