Cohen, Joel E.; Kemperman, Johannes H. B.; Zbăganu, Gheorghe H.
A family of inequalities originating from coding of messages. (English) Zbl 1139.15009

Summary: This paper presents 96 new inequalities with common structure, all elementary to state but many not elementary to prove. For example, if $n$ is a positive integer and $a = (a_1, \ldots, a_n)$ and $b = (b_1, \ldots, b_n)$ are arbitrary vectors $\mathbb{R}^n_+ = [0, \infty)^n$, and $\rho(m_{i,j})$ is the spectral radius of an $n \times n$ matrix with elements $m_{i,j}$, then

$$\sum_{i,j} \min((a_i,a_j),(b_i,b_j)) \leq \sum_{i,j} \min((a_i,b_j),(b_i,a_j)),$$

$$\sum_{i,j} \max((a_i + a_j),(b_i + b_j)) \geq \sum_{i,j} \max((a_i + b_j),(b_i + a_j)),$$

$$\rho(\min((a_i,a_j),(b_i,b_j))) \leq \rho(\min((a_i,b_j),(b_i,a_j))),$$

$$\sum_{i,j} \min((a_i,a_j),(b_i,b_j)) x_i x_j \leq \sum_{i,j} \min((a_i,b_j),(b_i,a_j)) x_i x_j,$$

for all real $x_i$, $i = 1, \ldots, n,$

$$\iint \log([(f(x) + f(y))(g(x) + g(y)))] d\mu(x) d\mu(y) \leq \iint \log([(f(x) + g(y))(g(x) + f(y)))] d\mu(x) d\mu(y).$$

The second inequality is obtained from the first inequality (which is due to G. Zbăganu [Proc. Rom. Acad., Ser. A, Math. Phys. Tech. Sci. Inf. Sci. 1, No. 1, 15–19 (2000; Zbl 1021.26020)]) by replacing min with max, and $\times$ with $+$, and by reversing the direction of the inequality. The third inequality is obtained from the first by replacing the summation by the spectral radius. The fourth inequality is obtained from the first by taking each summand as a coefficient in a quadratic form. The fifth inequality is obtained from the first by replacing both outer summations by products, min by $\times$, and by reversing the direction of the inequality. The proofs of these inequalities are mysteriously diverse.

A nice generalization of the first inequality is proved: Let $*$ be one of the four operations $+, \times$, min and max on an appropriate interval $J$ of $\mathbb{R}$. Let $a, b \in J^n$. Denote by $a* a$ the $n \times n$ matrix $a_{i,j} = a_i * a_j$. Then the matrix $a* a$ is more different from $b* b$ than $a* b$ is from $b* a$. Precisely, if $\|A\| = \sum_{i \leq j \leq n} |a_{i,j}|$, then $\|a* a - b* b\| \geq \|a* b - b* a\|$. 

MSC:

15A45 Miscellaneous inequalities involving matrices
39B62 Functional inequalities, including subadditivity, convexity, etc.
26D15 Inequalities for sums, series and integrals
94A15 Information theory (general)
60E15 Inequalities; stochastic orderings

Keywords:

information theory; operations research; Brownian bridge; (max,$+$)-algebra; quadratic form; spectral radius

Full Text: DOI

References:


4 Cuninghame-Green, R., Minimax algebra, Lecture notes in economics and mathematical systems 166, math. rev. 82a:90043, (1979.), Springer-Verlag


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