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Conservation laws with discontinuous flux: a short introduction. (English) Zbl 1138.35365

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Summary: Conservation laws with discontinuous flux have attracted recent attention both due to their numerous applications and the intriguing theoretical challenges raised by their well-posedness and numerical analysis. This introductory note states the basic problem considered in the eight contributions of this Special Issue. Three different types of applications are surveyed where these equations appear, motivated by spatially heterogeneous physical models, adjoint problems for parameter identification, and numerical methods for systems of conservation laws, respectively. Basic problems arising in the analysis of these equations are discussed, and the contributions of the Special Issue are presented.

MSC:

35L65 Hyperbolic conservation laws
35B30 Dependence of solutions to PDEs on initial and/or boundary data and/or on parameters of PDEs

Keywords:

Conservation laws; Discontinuous flux; Numerical methods; Transport equations; Well-posedness analysis

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References:

[1] Bressan A (2000). Hyperbolic systems of conservation laws: the one-dimensional Cauchy problem. Oxford University Press, Oxford, UK · Zbl 0997.35002

[2] Dafermos CM (2005). Hyperbolic conservation laws in continuum physics, 2nd edn. Springer Verlag, Berlin · Zbl 1078.35001

[3] Holden H and Risebro NH (2007). Front tracking for conservation laws. Second Corr. Printing, Springer Verlag, New York · Zbl 1239.35003

[4] Serre D (1999). Systems of conservation laws 1. Cambridge University Press, Cambridge, UK · Zbl 1063.35520

[5] Lighthill MJ and Whitham GB (1955). On kinematic waves. II. A theory of traffic flow on long crowded roads. Proc Roy Soc London Ser A 229: 317–345 · Zbl 0064.20906 · doi:10.1098/rspa.1955.0089

[6] Richards PI (1956). Shock waves on the highway. Oper Res 4: 42–51· doi:10.1287/opre.4.1.42

[7] Mochon S (1987). An analysis of the traffic on highways with changing surface conditions. Math Model 9: 1–11· doi:10.1016/0270-0255(87)90068-6

[8] Bürger R, Karlsen KH, Mishra S and Towers JD (2005). On conservation laws with discontinous flux. In: Wang, Y and Hutter, K (eds) Trends in applications of mathematics to mechanics, pp 75–84. Shaker Verlag, Aachen

[9] Ansorge R (1990). What does the entropy condition mean in traffic flow theory?. Transp Res B 24B: 133–143· doi:10.1016/0191-2615(90)90024-S

[10] Bürger R, García A, Karlsen KH, Towers JD (2008) Difference schemes, entropy solutions, and speedup impulse for an inhomogeneous kinematic traffic flow model. Netw Heterog Media (to appear)

[11] Kaasschieter EF (1999). Solving the Buckley-Leverett equation with gravity in a heterogeneous porous medium. Comput Geosci 3: 23–48 · Zbl 0952.76085 · doi:10.1023/A:1011574824970

[12] Van Duijn CJ, De Neef MJ and Molenaar J (1995). Effects of capillary forces on immiscible two-phase flow in strongly heterogeneous porous media. Transp Porous Media 21: 71–93· doi:10.1007/BF00615335

[13] Bürger R, Karlsen KH and Towers JD (2005). Closed-form and finite difference solutions to a population balance model of grinding mills. J Eng Math 51: 165–195 · Zbl 1097.74045 · doi:10.1007/s10665-004-1054-4

[14] Čanić S, Mirković D (2001) A hyperbolic system of conservation laws in modeling endovascular treatment of abdominal aortic aneurysm. Hyperbolic problems: theory, numerics, applications, vol I, II (Magdeburg, 2000). Int Ser Numer Math 140, 141:227–236

[15] Jin S and Wen X (2005). Hamiltonian-preserving schemes for the Liouville equation with discontinuous potentials. Comm Math Sci 3: 285–315 · Zbl 1094.35074

[16] Ross DS (1988). Two new moving boundary problems for scalar conservation laws. Comm Pure Appl Math 41: 725–737· doi:10.1002/cpa.3160410511

[17] Terracina A (1999). A free boundary problem for scalar conservation laws. SIAM J Math Anal 30: 985–1009 · Zbl 0936.35202 · doi:10.1137/S0036141097325307

[18] Ostrov DN (1999). Viscosity solutions and convergence of monotone schemes for synthetic aperture radar shape-from-shading equations with discontinuous intensities. SIAM J Appl Math 59: 2060–2085 · Zbl 0936.35048 · doi:10.1137/S0036139997327174

[19] Ostrov DN (2002). Solutions of Hamilton-Jacobi equations and scalar conservation laws with discontinuous space-time dependence. J Differential Equations 182: 51–77 · Zbl 1009.35015 · doi:10.1006/jdeq.2001.4088

[20] Garavello M and Piccoli B (2006). Traffic Flow on Networks. American Institute of Mathematical Sciences, Springfield, MO, USA · Zbl 1136.90012

[21] Herty M, Seaïd M and Singh AK (2007). A domain decomposition method for conservation laws with discontinuous flux function. Appl Numer Math 57: 361–373 · Zbl 1128.65078 · doi:10.1016/j.apnum.2006.04.003

[22] Holden H and Risebro NH (1995). A mathematical model of traffic flow on a network of unidirectional roads. SIAM J Math Anal 26: 999–1017 · Zbl 0833.35089 · doi:10.1137/S0036141093243289

[23] Bouchut F and James F (1998). One-dimensional transport equations with discontinuous coefficients. Nonlinear Anal 32: 891–933 · Zbl 0989.35130 · doi:10.1016/S0362-546X(97)00536-1

[24] Coronel A, James F and Sepúlveda M (2003). Numerical identification of parameters for a model of sedimentation processes. Inverse Problems 19: 951–972 · Zbl 1041.35079 · doi:10.1088/0266-5611/19/4/311

[25] James F and Sepúlveda M (1999). Convergence results for the flux identification in a scalar conservation law. SIAM J Control Optim 37: 869–891 · Zbl 0970.35161 · doi:10.1137/S0363012996272722

[26] Klingenberg C and Risebro NH (2001). Stability of a resonant system of conservation laws modeling polymer flow with gravitation. J Differential Equations 170: 344–380 · Zbl 0977.35083 · doi:10.1006/jdeq.2000.3826

[27] Adimurthi , Jaffré J and Veerappa Gowda GD (2004). Godunov-type methods for conservation laws with a flux function discontinuous in space. SIAM J Numer Anal 42: 179–208 · Zbl 1081.65082 · doi:10.1137/S003614290139562X

[28] Adimurthi , Mishra S and Veerappa Gowda GD (2005). Optimal entropy solutions for conservation laws with discontinuous flux functions. J Hyperbolic Differential Equations 2: 783–837 · Zbl 1093.35045 · doi:10.1142/S0219891605000622

[29] Audusse E and Perthame B (2005). Uniqueness for scalar conservation laws with discontinuous flux via adapted entropies. Proc Roy Soc Edinburgh Sect A 135: 253–265 · Zbl 1071.35079 · doi:10.1017/S0308210500003863

[30] Bachmann F and Vovelle J (2006). Existence and uniqueness of entropy solution of scalar conservation laws with a flux function involving discontinuous coefficients. Comm Partial Differential Equations 31: 371–395 · Zbl 1102.35064 · doi:10.1080/03605300500358095

[31] Diehl S (1995). On scalar conservation laws with point source and discontinuous flux function. SIAM J Math Anal 26: 1425–1451 · Zbl 0852.35094 · doi:10.1137/S0036141093242533

[32] Klingenberg C and Risebro NH (1995). Convex conservation laws with discontinuous coefficients, existence, uniqueness and asymptotic behavior. Comm Partial Differential Equations 20: 1959–1990 · Zbl 0836.35090 · doi:10.1080/03605309508821159

[33] Lyons WK (1982/1983). Conservation laws with sharp inhomogeneities. Quart Appl Math 40: 385–393

[34] Karlsen KH, Risebro NH, Towers JD (2003) L 1 stability for entropy solutions of nonlinear degenerate parabolic convection-diffusion equations with discontinuous coefficients. Skr K Nor Vid Selsk, 49 pp · Zbl 1036.35104

[35] Karlsen KH and Towers JD (2004). Convergence of the Lax-Friedrichs scheme and stability for conservation laws with a discontinuous space-time dependent flux. Chin Ann Math 25B: 287–318 · Zbl 1112.65085 · doi:10.1142/S0252959904000299

[36] Lin LW, Temple B and Wang JH (1995). Suppression of oscillations in Godunov's method for a resonant non-strictly hyperbolic system. SIAM J Numer Anal 32: 841–864 · Zbl 0830.35079 · doi:10.1137/0732038

[37] Towers JD (2000). Convergence of a difference scheme for conservation laws with a discontinuous flux. SIAM J Numer Anal 38: 681–698 · Zbl 0972.65060 · doi:10.1137/S0036142999363668

[38] Xu Z, Zhang P and Liu R (2007). {\(\delta\)}-mapping algorithm coupled with WENO reconstruction for nonlinear elasticity in heterogeneous media. Appl Numer Math 57: 103–116 · Zbl 1102.74047 · doi:10.1016/j.apnum.2006.01.003

[39] Zhang P and Liu R (2005). Generalization of Runge–Kutta discontinuous Galerkin method to LWR traffic model with inhomogeneous road conditions. Numer Meth Partial Differential Equations 21: 80–88 · Zbl 1067.65105 · doi:10.1002/num.20023

[40] Seaïd M (2006). Stable numerical methods for conservation laws with discontinuous flux function. Appl Math Comput 175: 383–400 · Zbl 1088.65080 · doi:10.1016/j.amc.2005.07.052