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An improved weighted essentially non-oscillatory scheme for hyperbolic conservation laws. (English) Zbl 1136.65076

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Summary: We develop an improved version of the classical fifth-order weighted essentially non-oscillatory finite difference scheme of *G.-S. Jiang* and *C.-W. Shu* [ibid. 126, No. 1, 202–228 (1996; Zbl 0877.65065)] (WENO-JS) for hyperbolic conservation laws. Through the novel use of a linear combination of the low order smoothness indicators already present in the framework of WENO-JS, a new smoothness indicator of higher order is devised and new non-oscillatory weights are built, providing a new WENO scheme (WENO-Z) with less dissipation and higher resolution than the classical WENO.

This new scheme generates solutions that are sharp as the ones of the mapped WENO scheme (WENO-M) of *A. K. Henrick, T. D. Aslam* and *J. M. Powers* [ibid. 207, No. 2, 542–567 (2005; Zbl 1072.65114)], however with a 25% reduction in CPU costs, since no mapping is necessary. We also provide a detailed analysis of the convergence of the WENO-Z scheme at critical points of smooth solutions and show that the solution enhancements of WENO-Z and WENO-M at problems with shocks comes from their ability to assign substantially larger weights to discontinuous stencils than the WENO-JS scheme, not from their superior order of convergence at critical points.

Numerical solutions of the linear advection of discontinuous functions and nonlinear hyperbolic conservation laws as the one dimensional Euler equations with Riemann initial value problems, the Mach 3 shock-density wave interaction and the blastwave problems are compared with the ones generated by the WENO-JS and WENO-M schemes. The good performance of the WENO-Z scheme is also demonstrated in the simulation of two dimensional problems as the shock-vortex interaction and a Mach 4.46 Richtmyer-Meshkov instability (RMI) modeled via the two dimensional Euler equations.

MSC:

65M06	Finite difference methods for initial value and initial-boundary value problems involving PDEs
35L65	Hyperbolic conservation laws
35L67	Shocks and singularities for hyperbolic equations
65M12	Stability and convergence of numerical methods for initial value and initial-boundary value problems involving PDEs

Keywords:

weighted essentially non-oscillatory; hyperbolic conservation laws; smoothness indicators; WENO weights; numerical examples; finite difference scheme; convergence; shocks; linear advection; Euler equations; shock-vortex interaction

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