Cannon, James W.; **Thurston, William P.**
Group invariant Peano curves. (English) Zbl 1136.57009
Geom. Topol. 11, 1315-1355 (2007).

Let S be a hyperbolic surface, whose universal cover is the hyperbolic plane $\mathbb{H}^2$. A discrete faithful representation of the fundamental group of S in the group $\mathrm{Isom}(\mathbb{H}^3)$ (or the image of such a representation) is called doubly degenerate if the limit set of the induced group action on the compactification $\mathbb{H}^3 \cup S^2_\infty$ is equal to the sphere S^2_∞.

One of the aims of this paper is to describe some doubly degenerate groups. The main result is that if M is a closed hyperbolic 3-manifold which fibers over the circle with pseudo-Anosov monodromy, then the lift of the inclusion map of the fiber S in M to the hyperbolic universal covers extends continuously to a map between the compactifications of the covering spaces, and induces at the boundary an equivariant S^2_∞-filling Peano curve. In this situation, S is a closed surface, and the authors conjecture that the result extends to the case where S is a once-punctured hyperbolic surface. Evidence for this conjecture is provided by the case of a figure-eight knot complement, which the authors analyze in detail.

The study of sphere-filling curves is based on a theorem by R. L. Moore which gives a condition under which the quotient of the 2-sphere by an equivalence relation induced by a cellular decomposition is homeomorphic to the 2-sphere. In the main example considered, the 2-sphere decomposition is obtained by collapsing two laminations.

The paper under review contains several fundamental ideas and techniques of 3-dimensional geometry and topology, and it has been circulated as a preprint for several years.

Reviewer: Athanase Papadopoulos (Strasbourg)

MSC:

57M60	Group actions on manifolds and cell complexes in low dimensions
57M50	General geometric structures on low-dimensional manifolds
57N05	Topology of the Euclidean 2-space, 2-manifolds (MSC2010)
57N60	Cellularity in topological manifolds
20F65	Geometric group theory

Keywords:

hyperbolic 3-manifold; figure-eight knot complement; invariant Peano curve; doubly degenerate group; pseudo-Anosov monodromy

Full Text: DOI

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