Obukhovskii, Valeri; Zecca, Pietro; Zvyagin, Victor
On some generalizations of the Landesman-Lazer theorem. (English) Zbl 1127.47056
Fixed Point Theory 8, No. 1, 69-85 (2007).

For a bounded domain $\Omega \subseteq \mathbb{R}^N$ with smooth boundary, and for $p \geq 2$ such that $2p > N$, consider a linear (unbounded) Fredholm operator A in $L^p(\Omega)$ with domain $D(A) = W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega)$. Here $W^{k,p}(\Omega)$ is the Sobolev space of functions with k distributional derivatives in $L^p(\Omega)$, and $W_0^{1,p}(\Omega)$ is the closure of $C_c^\infty(\Omega)$ in $W^{1,p}(\Omega)$. Suppose that A is symmetric with respect to the scalar product in $L^2(\Omega)$, and that $\ker A$ is the 1-dimensional space spanned by a function $\omega \in D(A) \setminus \{0\}$.

Let $g(r)$ be a given continuous function on $\mathbb{R}$ with existing limits $g(\pm\infty)$ as $r \to \pm\infty$, such that $g(-\infty) \leq g(r) \leq g(+\infty)$ for all r. Suppose that $\phi : \Omega \times \mathbb{R} \to \mathbb{R}$ has existing limits

$$\underline{\phi}(x,\xi) := \liminf_{\xi' \to \xi} \phi(x,\xi') \qquad \text{and} \qquad \overline{\phi}(x,\xi) := \limsup_{\xi' \to \xi} \phi(x,\xi')$$

for almost all $x \in \Omega$, and that $\underline{\phi}, \overline{\phi}$ are superpositionally measurable. Finally, suppose that there are $f_*, f^* \in L^p(\Omega)$ such that $f_*(x) \leq \phi(x,\xi) \leq f^*(x)$ for a.e. $x \in \Omega$ and all $\xi \in \mathbb{R}$.

Under the Landesman–Lazer type conditions

$$\int_{\omega>0} f^*\omega\,dx + \int_{\omega<0} f_*\omega\,dx < g(+\infty)\int_{\omega>0}\omega\,dx + g(-\infty)\int_{\omega<0}\omega\,dx,$$
$$\int_{\omega>0} f_*\omega\,dx + \int_{\omega<0} f^*\omega\,dx > g(-\infty)\int_{\omega>0}\omega\,dx + g(+\infty)\int_{\omega<0}\omega\,dx,$$

it is proved that the equation

$$(Au)(x) + g(u(x)) = \phi(x,u(x))$$

has a generalized solution. The same result is also proved for nonsymmetric A with higher dimensional kernel, albeit under abstract conditions in the functional setting.

The authors present an example involving a resonant nonlinearly oscillating membrane with a discontinuous obstacle, and and example on Lavrentiev's problem on detachable currents at the presence of resonance.

The proofs involve an application of coincidende index theory for multivalued maps.

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MSC:

47N20 Applications of operator theory to differential and integral equations
47J05 Equations involving nonlinear operators (general)
35R05 PDEs with low regular coefficients and/or low regular data
47H10 Fixed-point theorems
47H04 Set-valued operators

Keywords:

nonsmooth coefficients; coincidence index; multimap; multivalued nonlinear operator; Landesman-Lazer condition