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Standing waves for supercritical nonlinear Schrödinger equations. (English) Zbl 1124.35082
J. Differ. Equations 236, No. 1, 164-198 (2007).

The authors treat standing waves of the nonlinear Schrödinger equation in $\mathbb{R}^N$ with a supercritical nonlinearity, namely solutions of the problem

$$\Delta u - V(x)u + u^p = 0, \qquad u > 0, \lim_{|x|\to\infty} u(x) = 0. \tag{1}$$

Here it is assumed that V is bounded and nonnegative, $N \geq 3$, and $p > (N+2)/(N-2)$. To prove existence, in the case $N \geq 4$ and $p > (N+1)/(N-3)$, the only additional assumption on V is that of superquadratic decay at ∞. In the general supercritical case the authors have to assume a somewhat faster decay of V at ∞. Under these hypotheses it is proved that there exists a continuum of small solutions of (1).

This result stands in sharp contrast to the subcritical case, where one only expects solutions if V decays slower that quadratic at ∞. Moreover, it is remarkable that a continuum of solutions is presented without employing a singular perturbation parameter. The method rests on the existence of a scaled family $w_\lambda(x) = \lambda^{\frac{2}{p-1}} w(\lambda x)$ of radially symmetric positive solutions of the problem

$$\Delta w + w^p = 0 \qquad \text{in } \mathbb{R}^N.$$

By a fixed point argument it is shown that a solution of (1) exists near some $w_\lambda(\cdot - \xi)$ if λ is sufficiently small. For $p > (N+1)/(N-3)$ the center of symmetry ξ can be chosen arbitrarily within expanding domains of $\mathbb{R}^N$, essentially giving an $N+1$-dimensional set of solutions. In the general case ξ needs to be chosen from a fixed point set depending on λ.

Reviewer: Nils Ackermann (México, D.F.)

MSC:

35Q55 NLS equations (nonlinear Schrödinger equations)
37K40 Soliton theory, asymptotic behavior of solutions of infinite-dimensional Hamiltonian systems
35Q51 Soliton equations

Keywords:

Standing Waves; Nonlinear Schrödinger Equation; Supercritical Nonlinearity; Existence; Fixed point argument

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References:

[1] Ambrosetti, A.; Badiale, M.; Cingolani, S., Semiclassical states of nonlinear Schrödinger equations, Arch. ration. mech. anal., 140, 285-300, (1997) · Zbl 0896.35042

[2] Ambrosetti, A.; Felli, V.; Malchiodi, A., Ground states of nonlinear Schrödinger equations with potentials vanishing at infinity, J. eur. math. soc. (JEMS), 7, 1, 117-144, (2005) · Zbl 1064.35175

[3] Ambrosetti, A.; Malchiodi, A.; Ni, W.-M., Singularly perturbed elliptic equations with symmetry: existence of solutions concentrating on spheres, part I, Comm. math. phys., 235, 427-466, (2003) · Zbl 1072.35019

[4] Ambrosetti, A.; Malchiodi, A.; Ruiz, D., Bound states of nonlinear Schrödinger equations with potentials vanishing at infinity, J. anal. math., 98, 317-348, (2006) · Zbl 1142.35082

[5] Badiale, M.; D'Aprile, T., Concentration around a sphere for a singularly perturbed Schrödinger equation, Nonlinear anal., 49, 947-985, (2002) · Zbl 1018.35021

[6] Benci, V.; D'Aprile, T., The semiclassical limit of the nonlinear Schrödinger equation in a radial potential, J. differential equations, 184, 109-138, (2002) · Zbl 1060.35129

[7] Benci, V.; Cerami, G., Existence of positive solutions of the equation $- \operatorname{\Delta} u + a(x) u = u^{(N + 2)/(N - 2)}$ in $\mathbb{R}^N$, J. funct. anal., 88, 1, 90-117, (1990) · Zbl 0705.35042

[8] Benci, V.; Grisanti, C.R.; Micheletti, A.M., Existence of solutions for the nonlinear Schrödinger equation with $V(\infty) = 0$, (), 53-65 · Zbl 1231.35225

[9] Benci, V.; Grisanti, C.R.; Micheletti, A.M., Existence and non-existence of the ground state solution for the nonlinear Schrödinger equations with $V(\infty) = 0$, Topol. methods nonlinear anal., 26, 2, 203-219, (2005) · Zbl 1105.35112

[10] Byeon, J.; Wang, Z.-Q., Standing waves with a critical frequency for nonlinear Schrödinger equations, Arch. ration. mech. anal., 65, 295-316, (2002) · Zbl 1022.35064

[11] Cingolani, S.; Pistoia, A., Nonexistence of single blow-up solutions for a nonlinear Schrödinger equation involving critical Schrödinger exponent, Z. angew. math. phys., 55, 201-215, (2004) · Zbl 1120.35308

[12] Castro, R.; Felmer, P., Semi-classical limit for radial non-linear Schrödinger equation, Comm. math. phys., 256, 2, 411-435, (2005) · Zbl 1075.35010

[13] J. Dávila, M. del Pino, M. Musso, The supercritical Lane-Emden-Fowler equation in exterior domains, Comm. Partial Differential Equations, in press · Zbl 1137.35023

[14] J. Dávila, M. del Pino, M. Musso, J. Wei, Fast and slow decay solutions for supercritical elliptic problems in exterior domains, preprint · Zbl 1147.35030

[15] del Pino, M.; Felmer, P., Local mountain passes for semilinear elliptic problems in unbounded domains, Calc. var. partial differential equations, 4, 121-137, (1996) · Zbl 0844.35032

[16] del Pino, M.; Felmer, P., Semi-classical states for nonlinear Schrödinger equations, J. funct. anal., 149, 245-265, (1997) · Zbl 0887.35058

[17] del Pino, M.; Kowalczyk, K.; Wei, J., Nonlinear Schrödinger equations: concentration on weighted geodesics in the semi-classical limit, Comm. pure appl. math., 60, 1, 113-146, (2007)

[18] Floer, A.; Weinstein, A., Nonspreading wave packets for the cubic Schrödinger equation with a bounded potential, J. funct. anal., 69, 397-408, (1986) · Zbl 0613.35076

[19] Gui, C.; Ni, W.-M.; Wang, X., On the stability and instability of positive steady states of a semilinear heat equation in $\mathbb{R}^n$, Comm. pure appl. math., 45, 1153-1181, (1992) · Zbl 0811.35048

[20] Jeanjean, L.; Tanaka, K., Singularly perturbed elliptic problems with superlinear or asymptotically linear nonlinearities, Calc. var. partial differential equations, 21, 3, 287-318, (2004) · Zbl 1060.35012

[21] Kang, X.; Wei, J., On interacting bumps of semi-classical states of nonlinear Schrödinger equations, Adv. differential equations, 5, 7-9, 899-928, (2000) · Zbl 1217.35065

[22] Li, Y.Y., On a singularly perturbed elliptic equation, Adv. differential equations, 2, 6, 955-980, (1997) · Zbl 1023.35500

[23] Micheletti, A.M.; Musso, M.; Pistoia, A., Super-position of spikes for a slightly supercritical elliptic equation in $\mathbb{R}^N$, Discrete contin. dyn. syst., 12, 4, 747-760, (2005) · Zbl 1135.35036

[24] Norimichi, H.; Micheletti, A.M.; Pistoia, A., Existence of sign changing solutions for some critical problems on $\mathbb{R}^N$, Comm. pure appl. anal., 4, 1, 143-164, (2005) · Zbl 1123.35019

[25] Oh, Y.-G., Stability of semiclassical bound states of nonlinear Schrödinger equations with potentials, Comm. math. phys., 121, 11-33, (1989) · Zbl 0693.35132

[26] Rabinowitz, P.H., On a class of nonlinear Schrödinger equations, Z. angew. math. phys., 43, 270-291, (1992) · Zbl 0763.35087

[27] Souplet, P.; Zhang, Q., Stability for semilinear parabolic equations with decaying potentials in $\mathbb{R}^n$ and dynamical approach to the existence of ground states, Ann. inst. H. Poincaré anal. non linéaire, 19, 5, 683-703, (2002) · Zbl 1017.35033

[28] Wang, X., On concentration of positive bound states of nonlinear Schrödinger equations, Comm. math. phys., 153, 229-243, (1993) · Zbl 0795.35118