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Morse homology and manifolds with boundary. (English) Zbl 1123.58006

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From the author's introduction: Let M be an oriented compact manifold with boundary ∂M. We identify a collar neighborhood of the boundary with $[0,1)\times M$ and denote by r the standard coordinate on the first factor. Take a Riemannian metric g on $M\backslash\partial M$ such that $g|_{(0,1)\times\partial M}=\frac{1}{r}dr\otimes dr+rg_{\partial M}$, where $g_{\partial M}$ is a Riemannian metric on ∂M; we call such metrics horn end metrics. Let f be a Morse function on $M\backslash\partial M$ which satisfies the following conditions: (i) There is a Morse function $f_{\partial M}$ on ∂M such that $f|_{(0,1)\times\partial M}=rf_{\partial M}$. (ii) If γ is a critical point of $f_{\partial M}$, then $f_{\partial M}(\gamma)$ is not equal to zero.

We define Morse homology for such Morse functions; the complex is generated by the critical points of f and the positive ones of $f_{\partial M}$, and the boundary operator is defined by taking an algebraic count of the isolated descending gradient trajectories between critical points. We prove that the Morse homology is isomorphic to the singular homology of M, and construct the relative Morse homology sequence of ∂M in M. We remark that our Morse homology is a finite dimensional model of Floer homology for noncompact Lagrangian submanifolds properly embedded into a noncompact symplectic manifold with concave end; the collar neighborhoods correspond to concave ends, and Morse functions and the critical points of their differences correspond to Lagrangian submanifolds and their intersection points, which are the usual correspondences, and moreover boundary Morse functions and the critical points of their differences correspond to Legendrian submanifolds and the Reeb chords between them, and Morse homology of ∂M corresponds to contact homology.

Reviewer: Nils Ackermann (México, D.F.)

MSC:

58E05 Abstract critical point theory (Morse theory, Lyusternik-Shnirel'man theory, etc.) in infinite-dimensional spaces

55N35 Other homology theories in algebraic topology

57R70 Critical points and critical submanifolds in differential topology

53D40 Symplectic aspects of Floer homology and cohomology

Cited in **3** Documents

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Morse homology; manifold with boundary; horn end metric; concave end; Floer homology

Full Text: DOI

References:

[1] DOI: 10.2140/gt.2003.7.799 · Zbl 1131.53312 · doi:10.2140/gt.2003.7.799

[2] DOI: 10.1007/s002220200212 · Zbl 1029.57011 · doi:10.1007/s002220200212

[3] Eliashberg Y., Geom. Funct. Anal. Special Volume pp 560–

[4] Floer A., J. Differ. Geom. 28 pp 513– · Zbl 0674.57027 · doi:10.4310/jdg/1214442477

[5] Floer A., J. Differ. Geom. 30 pp 207– · Zbl 0678.58012 · doi:10.4310/jdg/1214443291

[6] DOI: 10.1007/BF02571639 · Zbl 0789.58022 · doi:10.1007/BF02571639

[7] DOI: 10.5802/afst.798 · Zbl 0859.57031 · doi:10.5802/afst.798

[8] DOI: 10.1007/978-3-642-62029-4· doi:10.1007/978-3-642-62029-4

[9] DOI: 10.1007/978-3-0348-8577-5· doi:10.1007/978-3-0348-8577-5

[10] Witten E., J. Differ. Geom. 17 pp 661– · Zbl 0499.53056 · doi:10.4310/jdg/1214437492