Segata, Jun-Ichi

Modified wave operators for the fourth-order nonlinear Schrödinger-type equation with cubic nonlinearity. (English) Zbl 1123.35072

Math. Methods Appl. Sci. 29, No. 15, 1785-1800 (2006).

The author considers the fourth-order nonlinear Schrödinger equation

$$i\partial_t u - \frac{1}{4}\partial_x^4 u = \lambda |u|^2 u$$

with cubic nonlinearity, a long range scattering problem where the solutions are not expected to be asymptotically free. Following the basic results in *T. Ozawa* [Commun. Math. Phys. 139, No. 3, 479–493 (1991; Zbl 0742.35043)], for final data $v(t)$ that are small in a weighted Sobolev space with mean zero, a unique global solution $u \in C(\mathbb{R}, L^2(\mathbb{R})) \cap L^8_{loc}(\mathbb{R}, L^\infty(\mathbb{R}))$ is constructed such that

$$\|u(t) - v(t)\|_{L^2_x(\mathbb{R})} = O(t^{-\alpha})$$

as $t \to \infty$, for all $\alpha \in (3/8, 1)$. The proof consists of two steps: The first step is an application of the contraction mapping principle to find the solution u near ∞, using Strichartz type estimates for the free evolution group $W(t)$. In contrast to the second-order problem, here no "MDFM"-decomposition is known for $W(t)$. This difficulty is overcome by using the method of stationary phase. In the second step global existence as $t \to -\infty$ is shown.

Reviewer: Nils Ackermann (México, D.F.)

MSC:

35Q55 NLS equations (nonlinear Schrödinger equations)
35P25 Scattering theory for PDEs
35B40 Asymptotic behavior of solutions to PDEs
81U05 2-body potential quantum scattering theory

Keywords:

long range scattering; higher-order dispersive equation; weighted Sobolev space; unique global solution; Strichartz type estimates; Strichartz type estimates

Full Text: DOI

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