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Concentration on curves for nonlinear Schrödinger equations. (English) Zbl 1123.35003

Commun. Pure Appl. Math. 60, No. 1, 113-146 (2007).

Suppose that $p > 1$ and $V\colon \mathbb{R}^N \to \mathbb{R}$ is smooth and such that $\inf_{x\in\mathbb{R}^N} V(x) > 0$. In the well known article [*A. Ambrosetti*, *A. Malchiodi*, and *W.-M. Ni*, Commun. Math. Phys. 235, No. 3, 427–466 (2003; Zbl 1072.35019)] it is proved that the problem

$$-\varepsilon^2\Delta u + V(x)u = u^p \tag{1}$$

possesses positive spike layer solutions concentrating near spheres of radius r_0 as $\varepsilon \to 0$ if V is radially symmetric and if the function $r \mapsto r^{N-1}V^\sigma(r)$ has a strict local extremum at r_0, with $\sigma := \frac{p+1}{p-1} - \frac{1}{2}$. Without imposing the condition of radial symmetry on V, the present work generalizes that result in the case $N = 2$ (and proves a related conjecture exposed in that article) to the existence of positive solutions of (1) concentrating near nondegenerate closed geodesics of the weighted metric $V^\sigma(dx_1^2 + dx_2^2)$ in $\mathbb{R}^2$. Here it has to be assumed that ε is small enough and satisfies a *gap condition*

$$|\varepsilon^2k^2 - \lambda_*| \geq c\varepsilon, \qquad \forall k \in \mathbb{N},$$

with some constants $c, \lambda_* > 0$. This condition implies bounds for the inverse of a linear differential operator that arises in the finite dimensional reduction.

Reviewer: Nils Ackermann (México, D.F.)

MSC:

35B25 Singular perturbations in context of PDEs
35J60 Nonlinear elliptic equations
35Q55 NLS equations (nonlinear Schrödinger equations)
47F05 General theory of partial differential operators

Keywords:

spike layer solution; semiclassical limit; standing wave

Full Text: DOI

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