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Fixed point indices and invariant periodic sets of holomorphic systems. (English)

Zbl 1121.32007

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The aim of the paper under review is to propose a qualitative study of the holomorphic ODE $\frac{dx}{dt} = F(x)$ with F being a holomorphic map from a neighborhood of O in \mathbb{C}^n to \mathbb{C}^n with only one isolated fixed point at O . The main result is that if dF_O has an eigenvalue $i\omega$ with $\omega \in \mathbb{R} \setminus \{0\}$ then the time $2\pi/\omega$ flow $\Phi(z)$ has O as an accumulation fixed point. Such a theorem is proved by looking at the fixed points index of F .

As a consequence of the previous theorem the author proves several interesting results. For instance, he proves that dF_O has a nonzero purely imaginary eigenvalue if and only if there exists a germ of complex variety of dimension at least one consisting of O and the periodic orbits of the same period of the system. Other results in terms of the length of the period and resonances of the pure imaginary eigenvalue are also given.

Reviewer: Filippo Bracci (Roma)

MSC:

- 32H50** Iteration of holomorphic maps, fixed points of holomorphic maps and related problems for several complex variables
32M25 Complex vector fields, holomorphic foliations, \mathbb{C} -actions
37C25 Fixed points and periodic points of dynamical systems; fixed-point index theory; local dynamics

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fixed point index; ODE; center of holomorphic dynamical systems

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