Pantev, Tony; **Sharpe, Eric**
GLSMs for gerbes (and other toric stacks). (English) Zbl 1119.14038
Adv. Theor. Math. Phys. 10, No. 1, 77-121 (2006).

The authors investigate gauged linear sigma model descriptions of toric stacks. This interesting paper includes (i) the gauged linear sigma model (GLSM) description of toric stacks; (ii) a description of the physics of GLSM; (iii) checking that physical predictions of those GLSM exactly match the corresponding stacks. The description of Deligne-Mumford stacks over toric varieties (toric stacks) is given with the help of stacky fans by *L. Borisov, L. Chen* and *G. Smith* ["The orbifold Chow ring of toric Deligne-Mumford stacks", J. Am. Math. Soc. 18, No. 1, 193–215 (2005; Zbl 1178.14057)]. It is well known that the GLSM is closely related to toric geometry. Some ground facts can be found in the book by [*K. Hori, S. Katz, A. Klemm, R. Pandharipande, R. Thomas, C. Vafa, R. Vakil*, and *E. Zaslow*, Mirror Symmetry. Clay Mathematics Monographs 1. Providence, RI: American Mathematical Society (AMS) (2003; Zbl 1044.14018)]. In the absence of a superpotential, the set of supersymmetric ground states of the GLSM is a toric variety. Conversely, toric varieties can be described as the set of ground states of an appropriate gauged linear sigma model.

The authors of the paper under review "discuss mirror symmetry for stacks, using Morrison-Plesser-Hori-Vafa techniques to compute mirrors explicitly", and also find a generalization of the mirror conjecture by *V. Batyrev* [J. Algebr. Geom. 3, 493–535 (1994; Zbl 0829.14023)] and by *L. Borisov* [Towards the Mirror Symmetry for Calabi-Yau Complete Intersections in Toric Fano Varieties, University of Michigan 1993 Preprint, alggeom/9310001].

After some introductory section, section two "Two-dimensional gauge theories with nonminimal charges" contains a brief review of the GLSM construction by *E. Witten* [Nucl. Phys. B 403, 159–222 (1993; Zbl 0910.14020)].

The third section of the paper under review considers gerbes over projective spaces, their presentations and quantum cohomology. Recall that the Picard groupoid on an open set U is the category whose objects are line bundles on U and whose morphisms are isomorphisms. A gerbe is a stack over a topological space which is locally isomorphic to the Picard groupoid of the space. A description of a stack or a gerbe in terms of an atlas with relations is known as a presentation. Every stack has a presentation of the form of a global quotient stack $[X/G]$, for some space X and some group G. However, presentations are not unique a given stack can have many different presentations of the form $[X/G]$. The authors investigated presentations of stacks in their preprint ["String compactifications on Calabi-Yau stacks", Nucl. Phys., B 733, No. 3, 233–296 (2006; Zbl 1119.81091)].

The fourth section concerns with stacks over Hirzebruch surfaces, analogue of flops, weighted projective stacks and quantum cohomology for toric stacks.

In section five they define Calabi-Yau gerbes as gerbes whose underlying variety is Calabi-Yau and discuss some of the gerbes. These include Landau-Ginzburg orbifolds, GLSM phases and expressions for D-termes.

Section six consists of the subsections: Toda theories corresponding to gerbes on projective spaces, mirrors to gerbes on the quintic, gerby minimal models, analog of Batyrev's mirror conjecture for stacks. In the section the authors review Batyrev's mirror construction for elliptic curves, propose a generalization of the construction and give examples of their applications.

In section seven conclusions are formulated. The appendix contains the description of toric stacks and gerbes by stacky fans.

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MSC:

14J32	Calabi-Yau manifolds (algebro-geometric aspects)
81T30	String and superstring theories; other extended objects (e.g., branes) in quantum field theory
14M25	Toric varieties, Newton polyhedra, Okounkov bodies
14J81	Relationships between surfaces, higher-dimensional varieties, and physics
81T40	Two-dimensional field theories, conformal field theories, etc. in quantum mechanics

Keywords:

toric stack; gauged linear sigma model; mirror symmetry

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