

Burq, Nicolas; Gérard, Patrick; Tzvetkov, Nikolay
Multilinear eigenfunction estimates and global existence for the three dimensional nonlinear Schrödinger equations. (English) [Zbl 1116.35109](#)
Ann. Sci. Éc. Norm. Supér. (4) 38, No. 2, 255-301 (2005).

From the abstract: "We study nonlinear Schrödinger equations, posed on a three dimensional Riemannian manifold M . We prove global existence of strong H^1 solutions on $M = S^3$ and $M = S^2 \times S^1$ as far as the nonlinearity is defocusing and sub-quintic and thus we extend results of Ginibre, Velo and Bourgain who treated the cases of the Euclidean space \mathbb{R}^3 and the torus $\mathbb{T}^3 = \mathbb{R}^3/\mathbb{Z}^3$ respectively. The main ingredient in our argument is a new set of multilinear estimates for spherical harmonics." Consider spherical harmonics H_p of order p on S^d , where d is the (arbitrary) dimension of the sphere S^d . The multilinear estimates mentioned above are of the form

$$\|H_p H_q\|_{L^2(S^d)} \leq C_1(d, p, q) \|H_p\|_{L^2(S^d)} \|H_q\|_{L^2(S^d)}$$

and

$$\|H_p H_q H_r\|_{L^2(S^d)} \leq C_2(d, p, q, r) \|H_p\|_{L^2(S^d)} \|H_q\|_{L^2(S^d)} \|H_r\|_{L^2(S^d)},$$

where $C_1(d, p, q)$ and $C_2(d, p, q, r)$ are given and shown to be asymptotically nearly optimal as $p, q, r \rightarrow \infty$.

Reviewer: Nils Ackermann (México, D.F.)

MSC:

35Q55 NLS equations (nonlinear Schrödinger equations)
35P05 General topics in linear spectral theory for PDEs
33C55 Spherical harmonics

Cited in **1** Review
Cited in **44** Documents

Keywords:

Nonlinear Schroedinger Equation; Eigenfunction Estimates; Global Existence; Compact Manifold

Full Text: [DOI](#) [arXiv](#) [Numdam](#) [Numdam](#) [EuDML](#)

References:

- [1] Bourgain J. , Fourier transform restriction phenomena for certain lattice subsets and application to nonlinear evolution equations I. Schrödinger equations , *Geom. Funct. Anal.* 3 (1993) 107 - 156 . MR 1209299 | Zbl 0787.35097 · Zbl 0787.35097 · doi:10.1007/BF01896020
- [2] Bourgain J. , Exponential sums and nonlinear Schrödinger equations , *Geom. Funct. Anal.* 3 (1993) 157 - 178 . Article | MR 1209300 | Zbl 0787.35096 · Zbl 0787.35096 · doi:10.1007/BF01896021
- [3] Bourgain J. , Eigenfunction bounds for the Laplacian on the n -torus , *Internat. Math. Res. Notices* 3 (1993) 61 - 66 . MR 1208826 | Zbl 0779.58039 · Zbl 0779.58039 · doi:10.1155/S1073792893000066
- [4] Bourgain J. , Remarks on Strichartz' inequalities on irrational tori, Personal communication, 2004.
- [5] Burq N. , Gérard P. , Tzvetkov N. , Strichartz inequalities and the nonlinear Schrödinger equation on compact manifolds , *Amer. J. Math.* 126 (3) (2004) 569 - 605 . MR 2058384 | Zbl 1067.58027 · Zbl 1067.58027 · doi:10.1353/ajm.2004.0016
- [6] Burq N. , Gérard P. , Tzvetkov N. , An instability property of the nonlinear Schrödinger equation on $\setminus(\{S\}^d\setminus)$, *Math. Res. Lett.* 9 (2-3) (2002) 323 - 335 . MR 1909648 | Zbl 1003.35113 · Zbl 1003.35113 · doi:10.4310/MRL.2002.v9.n3.a8
- [7] Burq N. , Gérard P. , Tzvetkov N. , The Cauchy problem for the nonlinear Schrödinger equation on compact manifold , *J. Nonlinear Math. Phys.* 10 (2003) 12 - 27 . MR 2063542 · Zbl 1362.35282
- [8] Burq N. , Gérard P. , Tzvetkov N. , Bilinear eigenfunction estimates and the nonlinear Schrödinger equation on surfaces , *Invent. Math.* 159 (2005) 187 - 223 . MR 2142336 | Zbl 1092.35099 · Zbl 1092.35099 · doi:10.1007/s00222-004-0388-x
- [9] Burq N. , Gérard P. , Tzvetkov N. , Multilinear estimates for Laplace spectral projectors on compact manifolds , *C. R. Acad. Sci. Paris Ser. I* 338 (2004) 359 - 364 . MR 2057164 | Zbl 1040.58011 · Zbl 1040.58011 · doi:10.1016/j.crma.2003.12.015
- [10] Cazenave T. , *Semi-Linear Schrödinger Equations* , Courant Lecture Notes in Mathematics , vol. 10 , New York University, American Mathematical Society , Providence, RI , 2003 . MR 2002047 | Zbl 1055.35003 · Zbl 1055.35003
- [11] Christ M., Colliander J., Tao T. , Ill-posedness for nonlinear Schrödinger and wave equations, Preprint, 2003. arXiv | MR

2018661 · [Zbl 1048.35101](#)

- [12] Gallot S. , Hulin D. , Lafontaine J. , Riemannian Geometry , Universitext, Springer-Verlag , Berlin , 1990 . MR 1083149 | [Zbl 0636.53001](#) · [Zbl 0636.53001](#)
- [13] Ginibre J. , Velo G. , On a class of nonlinear Schrödinger equations , J. Funct. Anal. 32 (1979) 1 - 71 . MR 533219 | [Zbl 0396.35029](#) · [Zbl 0396.35029](#) · [doi:10.1016/0022-1236\(79\)90077-6](#)
- [14] Ginibre J. , Le problème de Cauchy pour des EDP semi-linéaires périodiques en variables d'espace (d'après Bourgain) , Séminaire Bourbaki, Exp. 796, Astérisque 237 (1996) 163 - 187 . Numdam | MR 1423623 | [Zbl 0870.35096](#) · [Zbl 0870.35096](#)
- [15] Hörmander L. , The spectral function of an elliptic operator , Acta Math. 121 (1968) 193 - 218 . MR 609014 | [Zbl 0164.13201](#) · [Zbl 0164.13201](#) · [doi:10.1007/BF02391913](#)
- [16] Hörmander L. , Oscillatory integrals and multipliers on $\mathcal{F}(\mathbb{L}^p)$, Ark. Math. 11 (1973) 1 - 11 . MR 340924 | [Zbl 0254.42010](#) · [Zbl 0254.42010](#) · [doi:10.1007/BF02388505](#)
- [17] Kato T. , On nonlinear Schrödinger equations , Ann. Inst. Henri Poincaré Phys. Théor. 46 (1987) 113 - 129 . Numdam | MR 877998 | [Zbl 0632.35038](#) · [Zbl 0632.35038](#)
- [18] Klainerman S. , Machedon M. , Remark on Strichartz-type inequalities , Internat. Math. Res. Notices 5 (1996) 201 - 220 , With appendices by J. Bourgain and D. Tataru. MR 1383755 | [Zbl 0853.35062](#) · [Zbl 0853.35062](#) · [doi:10.1155/S1073792896000153](#)
- [19] Klainerman S. , Machedon M. , Finite energy solutions of the Yang-Mills equations in \mathbb{R}^{3+1} , Ann. of Math. (2) 142 (1) (1995) 39 - 119 . MR 1338675 | [Zbl 0827.53056](#) · [Zbl 0827.53056](#) · [doi:10.2307/2118611](#)
- [20] Koch H., Tataru D. , Personal communication, 2004.
- [21] Lions J.-L. , Quelques méthodes de résolution des équations aux dérivées partielles non linéaires , Dunod , Paris , 1969 . [Zbl 0189.40603](#) · [Zbl 0189.40603](#)
- [22] Sogge C. , Oscillatory integrals and spherical harmonics , Duke Math. J. 53 (1986) 43 - 65 . Article | MR 835795 | [Zbl 0636.42018](#) · [Zbl 0636.42018](#) · [doi:10.1215/S0012-7094-86-05303-2](#)
- [23] Sogge C. , Concerning the \mathcal{L}^p norm of spectral clusters for second order elliptic operators on compact manifolds , J. Funct. Anal. 77 (1988) 123 - 138 . MR 930395 | [Zbl 0641.46011](#) · [Zbl 0641.46011](#) · [doi:10.1016/0022-1236\(88\)90081-X](#)
- [24] Sogge C. , Fourier Integrals in Classical Analysis , Cambridge Tracts in Mathematics , 1993 . MR 1205579 | [Zbl 0783.35001](#) · [Zbl 0783.35001](#)
- [25] Stein E.M. , Harmonic Analysis: Real-Variable Methods, Orthogonality, and Oscillatory Integrals , Monographs in Harmonic Analysis , vol. III , Princeton University Press , Princeton, NJ , 1993 . MR 1232192 | [Zbl 0821.42001](#) · [Zbl 0821.42001](#)
- [26] Szegő G. , Orthogonal Polynomials , Colloq. Publications , American Mathematical Society , Providence, RI , 1974 . . [Zbl 0023.21505](#)
- [27] Zygmund A. , On Fourier coefficients and transforms of functions of two variables , Studia Math. 50 (1974) 189 - 201 . Article | MR 387950 | [Zbl 0278.42005](#) · [Zbl 0278.42005](#)
- [28] Tao T. , Multilinear weighted convolutions of \mathcal{L}^2 functions, and applications to non-linear dispersive equations , Amer. J. Math. 123 (2001) 839 - 908 . MR 1854113 | [Zbl 0998.42005](#) · [Zbl 0998.42005](#) · [doi:10.1353/ajm.2001.0035](#)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.