Deift, P.; **Its, A.**; **Krasovsky, I.**; **Zhou, X.**

The Widom–Dyson constant for the gap probability in random matrix theory. (English)

Zbl 1116.15019
J. Comput. Appl. Math. 202, No. 1, 26-47 (2007).

An asymptotic question in the theory of the Gaussian unitary ensemble of random matrices is considered. In the bulk scaling limit, the probability that there are no eigenvalues in the interval $(0, 2s)$ is given by $P_s = \det(I - K_s)$, where K_s is the trace-class operator with kernel $K_s(x, y) = \frac{\sin(x-y)}{\pi(x-y)}$ acting on $L^2(0, 2s)$.

The asymptotic behaviour of P_s as $s \to \infty$ was determined earlier and of particular interest in an asymptotic expansion is the Widom-Dyson constant $c_0 = \frac{1}{12} \ln 2 + 3\zeta'(-1)$, where $\zeta(z)$ is the Riemann zeta-function. A new derivation of this constant which does not rely on certain a priory information is presented. This approach has the potential advantage of being applicable to other problems involving the computation of critical constants, where a priori information may not be available.

Reviewer: Václav Burjan (Praha)

MSC:

15B52 Random matrices (algebraic aspects)

Cited in **27** Documents

Keywords:

random matrices; asymptotic expansions; correlation functions; Riemann-Hilbert problem; Gaussian unitary ensemble; trace-class operator; Widom-Dyson constant

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