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Building blocks for arbitrary high order discontinuous Galerkin schemes. (English)

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Summary: In this article we propose the use of the ADER methodology of solving generalized Riemann problems to obtain a numerical flux, which is high order accurate in time, for being used in the Discontinuous Galerkin framework for hyperbolic conservation laws. This allows direct integration of the semi-discrete scheme in time and can be done for arbitrary order of accuracy in space and time. The resulting fully discrete scheme in time does not need more memory than an explicit first order Euler time-stepping scheme. This becomes possible because of an extensive use of the governing equations inside the numerical scheme itself via the so-called Cauchy-Kowalewski procedure. We give an efficient algorithm for this procedure for the special case of the nonlinear two-dimensional Euler equations. Numerical convergence results for the nonlinear Euler equations results up to 8th order of accuracy in space and time are shown

MSC:

65M60 Finite element, Rayleigh-Ritz and Galerkin methods for initial value and initial-boundary value problems involving PDEs

76M20 Finite difference methods applied to problems in fluid mechanics

Keywords:

Discontinuous Galerkin finite elements; ADER approach; generalized Riemann problems; Cauchy-Kowalewski procedure

Software:

HE-E1GODF

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