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High order finite difference WENO schemes with the exact conservation property for the shallow water equations. (English) [Zbl 1114.76340](#)

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Summary: Shallow water equations with nonflat bottom have steady state solutions in which the flux gradients are nonzero but exactly balanced by the source term. It is a challenge to design genuinely high order accurate numerical schemes which preserve exactly these steady state solutions. In this paper we design high order finite difference WENO schemes to this system with such exact conservation property (C-property) and at the same time maintaining genuine high order accuracy. Extensive one and two dimensional simulations are performed to verify high order accuracy, the exact C-property, and good resolution for smooth and discontinuous solutions.

MSC:

76M20 Finite difference methods applied to problems in fluid mechanics

76B15 Water waves, gravity waves; dispersion and scattering, nonlinear interaction

65M06 Finite difference methods for initial value and initial-boundary value problems involving PDEs

Cited in **4** Reviews
Cited in **117** Documents

Keywords:

Shallow water equation; WENO scheme; High order accuracy; Source term; Conservation laws; C-property

Software:

[HLLE](#)

Full Text: [DOI](#)

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