van Benthem, J.; **Bezhanishvili, G.**; **ten Cate, B.**; **Sarenac, D.**
Multimodal logics of products of topologies. (English) Zbl 1113.03018
Stud. Log. 84, No. 3, 369-392 (2006).

If modal logics L_1, L_2 with modalities $\Box_1, \Box_2$ are determined by classes $\mathbb{F}_1, \mathbb{F}_2$ of Kripke frames, then $L_1 \times L_2$ is determined by the class of products $\mathbb{F}_1 \times \mathbb{F}_2 = \langle W_1 \times W_2, R_1, R_2 \rangle$ and is axiomatized (by D. Gabbay and V. Shekhtman, under suitable conditions) by Fusion $L_1 + L_2$ plus $com = \Box_1\Box_2 p \to \Box_2\Box_1 p$ and $chr = \Diamond_1\Box_2 p \to \Box_2\Diamond_1 p$. In the topological semantics, when $\Box$ is interpreted as the interior of a set, products $X \times Y$ not always validate com and chr. The authors prove that both X and Y being Alexandrov spaces (intersection of an arbitrary family of open sets is open) is sufficient. However, for rationals $\mathbb{Q}$, the logic of $\mathbb{Q} \times \mathbb{Q}$ is complete for the fusion $\mathrm{S4} + \mathrm{S4}$, hence much weaker than $\mathrm{S4} \times \mathrm{S4}$. A new completeness proof of S4 for $\mathbb{Q}$ is presented. The authors introduce and investigate new kinds of topologies, horizontal and vertical, which they call coordinate topologies. In the typical case of $\mathbb{R} \times \mathbb{R}$ a set A is horizontally open if it contains with each point (x, y) a horizontal interval $(a, b) \times \{y\}$ for $a < x < b$ and similarly for the vertical topology.

Reviewer: G. E. Mints (Stanford)

MSC:

03B45	Modal logic (including the logic of norms)	Cited in **4** Reviews
54B10	Product spaces in general topology	Cited in **15** Documents

Keywords:

product modalities; fusion of modal logics; product of modal logics; topological product; horizontal topology; topological semantics; vertical topology

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