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The critical velocity for vortex existence in a two-dimensional rotating Bose-Einstein condensate. (English) Zbl 1106.58009

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The wave function of a Bose-Einstein condensate that is confined in the direction of the rotation axis by a harmonic trap decouples and can be analyzed in a two-dimensional setting. In this case, minimizing a modified Gross-Pitaevskii functional on a weighted Sobolev space under the unit mass constraint, the authors give an asymptotic estimate of the critical angular velocity for the nucleation of vortices in the interior of the region occupied by the condensate. In addition, near the critical velocity several estimates on the energy and the shape of the minimizer, and on the location of its vortex are proved.

Reviewer: [Nils Ackermann \(México, D.F.\)](#)

MSC:

- [58E50](#) Applications of variational problems in infinite-dimensional spaces to the sciences
- [35Q55](#) NLS equations (nonlinear Schrödinger equations)
- [47J30](#) Variational methods involving nonlinear operators
- [82C10](#) Quantum dynamics and nonequilibrium statistical mechanics (general)

Cited in **48** Documents

Keywords:

[Bose-Einstein condensate](#); [Gross-Pitaevskii functional](#); [critical velocity](#); [harmonic trap](#)

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