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Convergence of the Ricci flow toward a soliton. (English) Zbl 1106.53045

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Introduction: Our goal in this paper is to prove the following theorem (Theorem 1.1.): Let $(g_{ij})_t = -2R_{ij} + \frac{1}{\tau} g_{ij}$ be a Ricci flow on a closed manifold M with uniformly bounded curvature operators and diameters for all $t \in [0,\infty)$. Assume also that some limit soliton is integrable. Then there is an 1-parameter family of diffeomorphisms $\phi(t)$, a unique soliton $h(t)$ and constants C, δ, t_0 such that $|\phi(t)^* g(t) - h(o)|_{k,\alpha} < Ce^{-\delta t}$, for all $t \in [t_0, \infty)$. Moreover, if $\psi(t)$ is a diffeomorphism such that $h(t) = \psi^* h(0)$, then $|(\phi\psi)^* g(t) - h(t)|_{C^0} < Ce^{-Ct}$.

The ideas for the proof of Theorem 1.1 have been inspired by those of *J. Cheeger* and *G. Tian* in [Invent. Math. 118, 493–571 (1994; Zbl 0814.53034)].

Outline of the proof of Theorem 1.1: In order to deal with this problem, we will first construct a gauge on time intervals of an arbitrary length, so that in the chosen gauge, the τ-flow equation becomes strongly parabolic. We will look at the solutions of a strictly parabolic equation. It will turn out that our metrics (in the right gauge) will satisfy a strictly parabolic equation that is almost linear and therefore their behavior is modeled on the behavior of the solutions of the linear equation. There are 3 types of solutions of our strictly parabolic equation: solutions that have an exponential growth; solutions that have an exponential decay; and solutions that change very slowly.

Roughly speaking, the integrability condition means that the solutions of a linearized deformation equation for solitons arise from a curve of metrics satisfying the same soliton equation. To deal with those slowly changing solutions, we will use the integrability condition to change the reference soliton metric so that at the end, we deal only with the cases of either a growth or decay. We will rule out the possibility of exponential growth, by using the fact that our flow sequentially converges toward solitons and by using the arguments similar to that established by *L. Simon* in [Ann. Math. (2) 118, 525–571 (1983; Zbl 0549.35071)] and also later used by Cheeger and Tian in [loc. cit.]. We will be left with the exponential decay which will allow us to continue our gauge up to infinity.

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MSC:

53C44	Geometric evolution equations (mean curvature flow, Ricci flow, etc.) (MSC2010)	Cited in **9** Documents
53C43	Differential geometric aspects of harmonic maps	

Keywords:

Ricci flow; soliton

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