**Ohshika, Kenichi**; **Miyachi, Hideki**

**On topologically tame Kleinian groups with bounded geometry.** (English) Zbl 1104.57009

Minsky, Yair (ed.) et al., Spaces of Kleinian groups. Proceedings of the programme 'Spaces of Kleinian groups and hyperbolic 3-manifolds', Cambridge, UK, July 21–August 15, 2003. Cambridge: Cambridge University Press (ISBN 0-521-61797-9/pbk). London Mathematical Society Lecture Note Series 329, 29-48 (2006).

Recently, in partial collaboration with J. Brock and R. Canary, Y. Minsky has proven Thurston's ending lamination conjecture. The conjecture states that a topologically tame hyperbolic 3-manifold $M$ is determined up to isometry by its homeomorphism type together with the conformal structure at infinity of the (non-cuspidal part of the) geometrically finite ends and the ending laminations of the geometrically infinite ends. In earlier work, Minsky [*Y. N. Minsky*, J. Am. Math. Soc. 7, No. 3, 539–588 (1994; Zbl 0808.30027), Geom. Topol. 4, 117–148 (2000; Zbl 0953.30027), Invent. Math. 146, No. 1, 143–192 (2001; Zbl 1061.37026)] had proven the conjecture in the case when $M$ has freely indecomposable fundamental group and bounded geometry, the latter condition being that there is a positive lower bound for the lengths of closed geodesics in $M$. In this paper, the authors show how to adapt Minsky's approach for the bounded geometry case to remove the hypothesis on the fundamental group. Since the Marden conjecture has now been proven independently by I. Agol, D. Calegari, D. Gabai, and S. Choi, the tameness assumption may also be removed. Two applications of the authors' generalized theorem are given in the paper. The first is a uniqueness result that shows, roughly speaking and omitting some additional technical hypotheses, that if two maps of $\partial M$ have the same limiting behavior on a conformal structure of $\partial M$, when the images of the structure under powers of the maps are uniformized by structures on $M$ and the limits are taken in $\mathrm{AH}(M)$, then the maps are homotopic as maps into $M$. The second application concerns the third bounded cohomology $H^3_b(G;\mathbb{R})$. *T. Soma* proved [Topology 36, No. 6, 1221–1246 (1997; Zbl 0881.57019)] that if $G$ is a closed surface group of genus at least 2 and one fixes a $\delta_0 > 0$, then the classes in $H^3_b(G;\mathbb{R})$ that are induced from fundamental cohomology classes of doubly degenerate hyperbolic 3-manifolds whose injectivity radii are bounded below by $\delta_0$ cannot accumulate in $H^3_b(G;\mathbb{R})$. This implies [*T. Soma*, The third bounded cohomology and Kleinian groups. S. Kojima (ed.), Topology and Teichmüller spaces. Proc. 37th Taniguchi Symp., Katinkulta, Finland, July 24–28, 1995, 265–277 (1996; Zbl 1104.55300)] that $H^3_b(G;\mathbb{R})$ has dimension equal to the cardinality of the continuum. By applying their theorem to the case of Schottky groups, the authors show that analogous results hold when $G$ is a free group of rank at least 2.

For the entire collection see [Zbl 1089.30004].

Reviewer: Darryl McCullough (Norman)

**MSC:**

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| 57M50 | General geometric structures on low-dimensional manifolds | Cited in **3** Documents |
| 30F40 | Kleinian groups (aspects of compact Riemann surfaces and uniformization) | |

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hyperbolic; 3-manifold; Kleinian; tame; indecomposable; lamination; ending; Masur; domain; bounded; geometry; cohomology; Schottky; free