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The Ambrosetti–Prodi problem for the p-Laplace operator. (English) Zbl 1101.35033

Commun. Partial Differ. Equations 31, No. 4-6, 849-865 (2006).

The authors consider the quasilinear boundary value problem

$$-\Delta_p u = f(u) + t\phi(x) + h(x) \tag{1}$$

on a bounded domain $\Omega \subseteq \mathbb{R}^N$ with homogeneous Dirichlet boundary conditions. Here Δ_p denotes the p-Laplacian for $p > 1$, $\phi, h \in L^\infty(\Omega)$, and ϕ is positive. The function f is asymptotically p-linear, with different coefficients at $-\infty$ and ∞ ("jumping nonlinearity"). Inspired by *H. Brezis* and *L. Nirenberg* [C. R. Acad. Sci., Paris, Sér. I 317, No. 5, 465–472 (1993; Zbl 0803.35029)] it is proved that there exist $t_* \le t^*$ such that (1) admits at least two solutions for $t < t_*$, at least one solution for $t \le t^*$, and no solution for $t > t^*$. If $t < t_*$ the existence of one solution follows from the existence of a sub- and a supersolution. A second solution is found by degree arguments. In order to apply degree theory the authors prove a strong comparison principle for solutions of (1). If $p \ge 2$ then conditions are given to ensure that $t_* = t^*$.

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MSC:

35J65	Nonlinear boundary value problems for linear elliptic equations
35J60	Nonlinear elliptic equations
35J25	Boundary value problems for second-order elliptic equations
47H07	Monotone and positive operators on ordered Banach spaces or other ordered topological vector spaces
47H11	Degree theory for nonlinear operators
58E07	Variational problems in abstract bifurcation theory in infinite-dimensional spaces

Keywords:

Comparison principles for the p-Laplacian; Leray-Schauder degree; sub- and supersolutions; jumping nonlinearity

Full Text: DOI

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