Umarov, Sabir; Gorenflo, Rudolf

Cauchy and nonlocal multi-point problems for distributed order pseudo-differential equations. I. (English) Zbl 1100.35132

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The authors consider a general pseudo-differential problem with distributed order which arises in kinetic theory and in elasticity theory. Namely, they construct a representation formula for solutions of the Cauchy problem

$$\int_0^m A(r;D)D_*^r u(t,x)\,dr = B(D)u(t,x), \quad t>0,\ x\in\mathbb{R}^n,$$

$$\frac{\partial^k u(0,x)}{\partial t^k} = \varphi_k(x), \quad x\in\mathbb{R}^n,\ k=0,\dots,m-1,$$

where $D=(D_1,\dots,D_n)$, $D_j=-i\frac{\partial}{\partial x_j}$, $j=1,\dots,n$, and $A(r;D)$ and $B(D)$ are pseudo-differential operators. Then, they prove existence and uniqueness of strong and weak solutions in an appropriate space.

As pointed out in the paper, an essential feature of this model is that an integration is performed over the order of differentiation and this problem generalizes many existing ones in the literature such as $D_*^\beta u(t,x)=B(D)u(t,x)$, $D_*^\beta u(t,x)=\Delta u(t,x)$, $\frac{\partial u}{\partial t}(t,x)=D_0^\alpha u(t,x)$, $D_*^\beta u(t,x)=D_0^\alpha u(t,x)$, $\int_0^2 k(q)D^q y(t)\,dq + F(y)=f(t)$, ...etc. Finally, the authors discuss the more general case (multi-point value problem) of the equation in the problem with the condition

$$\sum_{j=0}^{m-1}\Gamma_{k_j}(D)\frac{\partial^k u(t_{k_j},x)}{\partial t^k} = \varphi_k(x), \quad x\in\mathbb{R}^n,\ k=0,\dots,m-1$$

and outline the related results.

Reviewer: Nasser-eddine Tatar (Dhahran)

MSC:

35S15 Boundary value problems for PDEs with pseudodifferential operators
26A33 Fractional derivatives and integrals
45K05 Integro-partial differential equations
35A05 General existence and uniqueness theorems (PDE) (MSC2000)
35S10 Initial value problems for PDEs with pseudodifferential operators

Keywords:

distributed order fractional differential equation; Cauchy problem; Caputo fractional derivative; existence; uniqueness; strong and weak solutions