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Transition from the annealed to the quenched asymptotics for a random walk on random obstacles. (English) Zbl 1099.82003

Ann. Probab. 33, No. 6, 2149-2187 (2005).

In the paper a natural transition mechanism describing the passage from a quenched regime to an annealed one, for a symmetric random walk on random obstacles on sites having an identical and independent law, is studied. An argument of the transition mechanism used in the paper was firstly proposed in [*B. Arous, L. Bogachev* and *S. Molchanov*, Probab. Theory Relat. Fields 132, 579–612 (2005; Zbl 1073.60017)].

Let $p(x,t)$ be the survival probability at time t of the random walk, starting from site x, and let $L(t)$ be some increasing function of time. In the paper it is studied the averaged quantity $p^{L(t)}(0,t) = \frac{1}{|\Lambda_{L(t)}|}\sum_{x\in\Lambda_{L(t)}} p(x,t)$, where $\Lambda_{L(t)} = [-(2L(t)+1,(2L(t)+1)]^d \cap \mathbb{Z}^d$.

It is shown that $p^{L(t)}(0,t)$ has different asymptotic behaviors depending on $L(t)$. Namely, there are constants $0<\gamma_1<\gamma_2$ such that if $L(t)\geq e^{\gamma t^{d/(d+2)}}$, with $\gamma>\gamma_1$, a law of large numbers is satisfied and the empirical survival probability decreases like the annealed one; if $L(t)\geq e^{\gamma t^{d/(d+2)}}$, with $\gamma>\gamma_2$, also a central limit theorem is satisfied. If $L(t)\ll t$, $p^{L(t)}(0,t)$ decreases like the quenched survival probability. If $t\ll L(t)$ and $\log L(t)\ll t^{d/(d+2)}$ an intermediate regime is obtained.

Furthermore, when the dimension $d=1$ it is possible to describe the fluctuations of $p^{L(t)}(0,t)$ when $L(t)=e^{\gamma t^{d/(d+2)}}$ with $\gamma<\gamma_2$: it is shown that they are infinitely divisible laws with a Lévy spectral function which explodes when $x\to 0$ as stable laws of characteristic exponent $\alpha<2$. These results show that the quenched and annealed survival probabilities correspond to a low- and high-temperature behavior of a mean-field type phase transition mechanism.

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MSC:

82B41 Random walks, random surfaces, lattice animals, etc. in equilibrium statistical mechanics

82B44 Disordered systems (random Ising models, random Schrödinger operators, etc.) in equilibrium statistical mechanics

60J45 Probabilistic potential theory

60J65 Brownian motion

82C22 Interacting particle systems in time-dependent statistical mechanics

Keywords:

parabolic Anderson model; random walk; enlargement of obstacles; principal eigenvalue; Wiener sausage

Full Text: DOI arXiv

References:

[1] Antal, P. (1994). Trapping problems for the simple random walk. Ph.D. thesis, ETH, Zürich.

[2] Antal, P. (1995). Enlargement of obstacles for the simple random walk. Ann. Probab. 23 1061–1101. JSTOR: · Zbl 0839.60064 · doi:10.1214/aop/1176988174

[3] Ben Arous, G., Bogachev, L. and Molchanov, S. (2005). Limit theorems for sums of random exponentials. Probab. Theory Related Fields 132 579–612. · Zbl 1073.60017 · doi:10.1007/s00440-004-0406-3

[4] Donsker, M. and Varadhan, S. R. S. (1975). Asymptotics for the Wiener sausage. Comm. Pure Appl. Math. 28 525–565. · Zbl 0333.60077 · doi:10.1002/cpa.3160280406

[5] Donsker, M. and Varadhan, S. R. S. (1979). On the number of distinct sites visited by a random walk. Comm. Pure Appl. Math. 32 721–747. · Zbl 0418.60074 · doi:10.1002/cpa.3160320602

[6] Gärtner, J. and Molchanov, S. A. (1990). Parabolic problems for the Anderson model. I. Intermittency and related topics. Comm. Math. Phys. 132 613–655. · Zbl 0711.60055 · doi:10.1007/BF02156540

[7] Gärtner, J. and Molchanov, S. A. (1998). Parabolic problems for the Anderson model. II. Second-order asymptotics and structure of high peaks. Probab. Theory Related Fields 111 17–55. · Zbl 0909.60040 · doi:10.1007/s004400050161

[8] Petrov, V. (1996). Limit Theorems of Probability Theory . Oxford Science Publications. · Zbl 0826.60001

[9] Pisztora, A. and Povel, T. (1999). Large deviation principle for random walk in a quenched random environment in the low speed regime. Ann. Probab. 27 1389–1413. · Zbl 0964.60056 · doi:10.1214/aop/1022677453

[10] Pisztora, A., Povel, T. and Zeitouni, O. (1999). Precise large deviation estimates for a one-dimensional random walk in a random environment. Probab. Theory Related Fields 113 191–219. · Zbl 0922.60059 · doi:10.1007/s004400050206

[11] Smoluchowski, M. V. (1918). Versuch einer mathemathischen Theorie der koagulations-kinetischen Lösungen. Z. Phys. Chem. 92 129–168.

[12] Sznitman, A. S. (1998). Brownian Motion Obstacles and Random Media. Springer, Berlin. · Zbl 0973.60003