Martel, Yvan; Merle, Frank; Tsai, Tai-Peng

Stability in H^1 of the sum of K solitary waves for some nonlinear Schrödinger equations.

(English) Zbl 1099.35134
Duke Math. J. 133, No. 3, 405-466 (2006).

Summary: We consider nonlinear Schrödinger (NLS) equations in $\mathbb{R}^d$ for $d = 1, 2$, and 3. We consider nonlinearities satisfying a flatness condition at zero and such that solitary waves are stable. Let $R_k(t, x)$ be K solitary wave solutions of the equation with different speeds $v_1, v_2, \dots, v_K$. Provided that the relative speeds of the solitary waves $v_k - v_{k-1}$ are large enough and that no interaction of two solitary waves takes place for positive time, we prove that the sum of the $R_k(t)$ is stable for $t \geq 0$ in some suitable sense in H^1.

To prove this result, we use an energy method and a new monotonicity property on quantities related to momentum for solutions of the nonlinear Schrödinger equation. This property is similar to the L^2 monotonicity property that has been proved by *Y. Martel* and *F. Merle* [J. Math. Pures Appl. (9) 79, No. 4, 339–425 (2000; Zbl 0963.37058)] for the generalized Korteweg-de Vries (gKdV) equations and that was used to prove the stability of the sum of K solitons of the gKdV equations by the authors of the present article [Commun. Math. Phys. 231, 347–373 (2002; Zbl 1017.35098)].

MSC:

35Q55 NLS equations (nonlinear Schrödinger equations)
37K45 Stability problems for infinite-dimensional Hamiltonian and Lagrangian systems
35Q51 Soliton equations
35B35 Stability in context of PDEs

Keywords:

NLS equations; energy method; monotonicity property; stability

Full Text: DOI Euclid

References:

[1] H. Berestycki and P.-L. Lions, Nonlinear scalar field equations, I: Existence of a ground state , Arch. Rational Mech. Anal. 82 (1983), 313–345. · Zbl 0533.35029

[2] V. S. Buslaev and G. S. Perelman, "On the stability of solitary waves for nonlinear Schrödinger equations" in Nonlinear Evolution Equations , Amer. Math. Soc. Transl. Ser. 2 164 , Amer. Math. Soc., Providence, 1995, 75–98. · Zbl 0841.35108

[3] T. Cazenave and P.-L. Lions, Orbital stability of standing waves for some nonlinear Schrödinger equations , Comm. Math. Phys. 85 (1982), 549–561. · Zbl 0513.35007 · doi:10.1007/BF01403504

[4] S. Cuccagna, Stabilization of solutions to nonlinear Schrödinger equations, Comm. Pure Appl. Math. 54 (2001), 1110–1145. · Zbl 1031.35129 · doi:10.1002/cpa.1018

[5] K. El Dika, Asymptotic stability of solitary waves for the Benjamin-Bona-Mahony equation , Discrete Contin. Dyn. Syst. 13 (2005), 583–622. · Zbl 1083.35019 · doi:10.3934/dcds.2005.13.583

[6] K. El Dika and Y. Martel, Stability of \(N\)-solitary waves for the generalized BBM equations , Dyn. Partial Differ. Equ. 1 (2004), 401–437. · Zbl 1080.35116 · doi:10.4310/DPDE.2004.v1.n4.a3

[7] J. Ginibre and G. Velo, On a class of nonlinear Schrödinger equations, I: The Cauchy problem, general case , J. Funct. Anal. 32 (1979), 1–32. · Zbl 0396.35028 · doi:10.1016/0022-1236(79)90076-4

[8] M. Grillakis, J. Shatah, and W. Strauss, Stability theory of solitary waves in the presence of symmetry , I , J. Funct. Anal. 74 (1987), 160–197. · Zbl 0656.35122 · doi:10.1016/0022-1236(87)90044-9

[9] P.-L. Lions, The concentration-compactness principle in the calculus of variations. The locally compact case, I , Ann. Inst. H. Poincaré Anal. Non Linéaire 1 (1984), 109–145. · Zbl 0541.49009

[10] J. H. Maddocks and R. L. Sachs, On the stability of KdV multi-solitons , Comm. Pure Appl. Math. 46 (1993), 867–901. · Zbl 0795.35107 · doi:10.1002/cpa.3160460604

[11] Y. Martel, Asymptotic \(N\)-soliton-like solutions of the subcritical and critical generalized Korteweg –.de Vries equations , Amer. J. Math. 127 (2005), 1103–1140. · Zbl 1090.35158 · doi:10.1353/ajm.2005.0033

[12] Y. Martel and F. Merle, A Liouville theorem for the critical generalized Korteweg –.de Vries equation , J. Math. Pures Appl. (9) 79 (2000), 339–425. · Zbl 0963.37058 · doi:10.1016/S0021-7824(00)00159-8

[13] -, Asymptotic stability of solitons for subcritical generalized KdV equations , Arch. Ration. Mech. Anal. 157 (2001), 219–254. · Zbl 0981.35073 · doi:10.1007/s002050100138

[14] -, Instability of solitons for the critical generalized Korteweg –.de Vries equation , Geom. Funct. Anal. 11 (2001), 74–123. · Zbl 0985.35071 · doi:10.1007/PL00001673

[15] Y. Martel, F. Merle, and T.-P. Tsai, Stability and asymptotic stability in the energy space of the sum of \(N\) solitons for subcritical gKdV equations , Comm. Math. Phys. 231 (2002), 347–373. · Zbl 1017.35098 · doi:10.1007/s00220-002-0723-2

[16] F. Merle, Existence of blow-up solutions in the energy space for the critical generalized KdV equation , J. Amer. Math. Soc. 14 (2001), 555–578. JSTOR: · Zbl 0970.35128 · doi:10.1090/S0894-0347-01-00369-1

[17] R. L. Pego and M. I. Weinstein, Asymptotic stability of solitary waves , Comm. Math. Phys. 164 (1994), 305–349. · Zbl 0805.35117 · doi:10.1007/BF02101705

[18] G. Perelman, “Some results on the scattering of weakly interacting solitons for nonlinear Schrödinger equations” in Spectral Theory, Microlocal Analysis, Singular Manifolds , Math. Top. 14 , Akademie, Berlin, 1997, 78–137. · Zbl 0931.35164

[19] -, Asymptotic stability of multi-soliton solutions for nonlinear Schrödinger equations , Comm. Partial Differential Equations 29 (2004), 1051–1095. · Zbl 1067.35113 · doi:10.1081/PDE-200033754

[20] I. Rodnianski, W. Schlag, and A. D. Soffer, Asymptotic stability of \(N\)-soliton states of NLS , to appear in Comm. Pure Appl. Math. · Zbl 1130.81053

[21] M. I. Weinstein, Modulational stability of ground states of nonlinear Schrödinger equations , SIAM J. Math. Anal. 16 (1985), 472–491. · Zbl 0583.35028 · doi:10.1137/0516034

[22] -, Lyapunov stability of ground states of nonlinear dispersive evolution equations , Comm. Pure Appl. Math. 39 (1986), 51–67. · Zbl 0594.35005 · doi:10.1002/cpa.3160390103

[23] V. E. Zakharov and A. B. Shabat, Exact theory of two-dimensional self-focusing and one-dimensional self-modulation of waves in nonlinear media , Soviet Physics JETP 34 (1972), 62–69.