Nucci, M. C.; **Leach, P. G. L.**; **Andriopoulos, K.**

Lie symmetries, quantisation and c-isochronous nonlinear oscillators. (English)

Zbl 1097.81042
J. Math. Anal. Appl. 319, No. 1, 357-368 (2006).

A classical Hamiltonian system is usually quantized by applying a quantization rule to the Hamiltonian governing the system. Among these rules the Weyl quantization scheme is often used because it always yields an L^2-symmetric Schrödinger operator.

In the papers by *F. V. Calogero* and *S. V. Graffi* [Phys. Lett., A 313, No. 5–6, 356–362 (2003; Zbl 1038.81014)], *F. V. Calogero* [Phys. Lett., A 319, No. 3–4, 240–245 (2003; Zbl 1038.81013)], and *F. V. Calogero* [J. Nonlinear Math. Phys. 11, 1–6 (2004; Zbl 1038.81015)] certain Hamiltonians of nonlinear oscillators were studied that depend on a parameter c in such a way that the solutions are c-isochronous. It was shown that under the Weyl quantization scheme, contrary to physical expectation the ground state energy depends on c, and that it also changes if certain nonlinear autonomous transformations of position and momentum are applied to the classical system. Starting with the simpler c-isochronous Hamiltonian

$$H = \frac{1}{2}\left(\frac{p^2}{c} + c\omega^2 q^2\right)$$

and its canonical quantization

$$2ic\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial x^2} - c^2\omega^2 x^2 u = 0 \tag{1}$$

the authors observe that here the ground state energy is independent of c. With respect to the same transformations of position and momentum as considered in the articles mentioned above they show that the ground state energies of the associated standard Weyl quantizations are different and *do* depend on c.

To remedy this deviation from physical expectation they devise a nonstandard quantization scheme that yields Schrödinger equations with the same c-independent ground state energies as (1) possesses. This scheme transforms (1) directly into different equations instead of quantizing the transformed Hamiltonians. Another feature of these nonstandard quantizations is that the number of Lie point symmetries is preserved across the transformations of position and momentum.

The authors argue that their results provide an incentive to search for nonstandard quantization procedures that preserve the number of Lie point symmetries, in order to obtain equations consistent with physical expectations. They do not claim to have provided such a scheme in general.

Reviewer: Nils Ackermann (México, D.F.)

MSC:

81S10 Geometry and quantization, symplectic methods
34C14 Symmetries, invariants of ordinary differential equations
34C15 Nonlinear oscillations and coupled oscillators for ordinary differential equations

Keywords:

Quantization; Lie Symmetries; Consistency

Full Text: DOI

References:

[1] Calogero, F.; Graffi, S., On the quantisation of a nonlinear Hamiltonian oscillator, Phys. lett. A, 313, 356-362, (2003) · Zbl 1038.81014
[2] Calogero, F., On the quantisation of two other nonlinear harmonic oscillators, Phys. lett. A, 319, 240-245, (2003) · Zbl

1038.81013

[3] Calogero, F., On the quantisation of yet another two nonlinear harmonic oscillators, J. nonlinear math. phys., 11, 1-6, (2004) · Zbl 1038.81015

[4] Leach, P.G.L., The solution of some quantum nonlinear oscillators with the common symmetry group $\mathit{SL}(2, R)$, J. phys. A, 38, 1543-1552, (2005) · Zbl 1065.81043

[5] Ermakov, V., Second order differential equations. conditions of complete integrability, Univ. izv. kiev ser. III, 9, 1-25, (1880), (trans. A.O. Harin)

[6] Pinney, E., The nonlinear differential equation $y'' + p(x) y + c y^{-3} = 0$, Proc. amer. math. soc., 1, 681, (1950) · Zbl 0038.24303

[7] Lemmer, R.L.; Leach, P.G.L., A classical viewpoint on quantum chaos, Arab. J. math. sci., 5, 1-17, (1999) · Zbl 0956.70015

[8] Camiz, P.; Geradi, A.; Marchioro, C.; Presutti, E.; Scacciatelli, E., Exact solution of a time-dependent quantal harmonic oscillator with a singular perturbation, J. math. phys., 12, 2040-2043, (1971) · Zbl 0224.35081

[9] Wolf, K.B., Integral transforms in science and engineering, (1979), Plenum New York · Zbl 0409.44001