

Lecuire, Cyril

Pleating of hyperbolic 3-manifolds. (Plissage des variétés hyperboliques de dimension 3.)
(English) [Zbl 1097.57017](#)
Invent. Math. 164, No. 1, 85-141 (2006).

The paper under review characterizes the measured laminations that occur as bending laminations of geometrically finite hyperbolic three-manifolds that are not fuchsian and nonelementary. In case that the boundary is incompressible or the lamination is a multicurve, such a characterization was obtained by *F. Bonahon* and *J.-P. Otal* in [*Ann. Math. (2)* 160, No. 3, 1013–1055 (2005; [Zbl 1083.57023](#))].

The characterization is given by three properties. First, the weight (the transverse measure) of each closed leaf has to be at most π . Secondly, there is an annularity condition: the intersection of the lamination with the boundary of every essential annulus is bounded uniformly away from zero. And thirdly, the intersection of the lamination with the boundary of every compressing disk is greater than 2π .

As in the work of Bonahon and Otal, the closure lemma is the main technical tool and is used in a convergence argument. However, in the general case proved here, the proof of the closure lemma is substantially more involved, and requires tools as a lemma of approximation for slightly pleated curves. The result was announced in the author's paper [*Sémin. Théor. Spectr. Géom.* 21, 103–115 (2003; [Zbl 1055.57022](#))], where the proof was also sketched.

Reviewer: [Joan Porti \(Bellaterra\)](#)

MSC:

- [57M50](#) General geometric structures on low-dimensional manifolds
- [57N10](#) Topology of general 3-manifolds (MSC2010)
- [30F40](#) Kleinian groups (aspects of compact Riemann surfaces and uniformization)
- [30F45](#) Conformal metrics (hyperbolic, Poincaré, distance functions)

Cited in 14 Documents

Keywords:

[measured geodesic lamination](#); [Kleinian group](#); [geometrically finite](#); [bending lamination](#)

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