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**The discontinuous Galerkin method with Lax–Wendroff type time discretizations.** (English)

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**Summary:** We develop a Lax–Wendroff time discretization procedure for the discontinuous Galerkin method (LWDG) to solve hyperbolic conservation laws. This is a method for time discretization alternative to the popular total variation diminishing Runge-Kutta time discretizations. The LWDG is a one-step, explicit, high-order finite element method. The limiter is performed once at every time step. As a result, LWDG is more compact than Runge-Kutta discontinuous Galerkin method, and the Lax–Wendroff time discretization procedure is more cost effective than Runge-Kutta time discretizations for certain problems including two-dimensional Euler systems of compressible gas dynamics when nonlinear limiters are applied.

**MSC:**

**76M10** Finite element methods applied to problems in fluid mechanics

**76N15** Gas dynamics (general theory)

Cited in **2** Reviews  
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**Keywords:**

Runge-Kutta method; explicit high-order finite element method; compressible gas dynamics; nonlinear limiters

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