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On orientability and degree of Fredholm maps. (English) Zbl 1093.58003

Mich. Math. J. 53, No. 2, 419-428 (2005).

To construct a meaningful $\mathbb{Z}$-valued degree theory for smooth maps between Banach manifolds whose differentials are Fredholm of index 0, some concept of orientability has to be introduced. Instead of requiring the manifolds to be orientable, it has been shown in [*P. M. Fitzpatrick*, *J. Pejsachowicz* and *P. J. Rabier*, J. Funct. Anal. 124, 1–39 (1994; Zbl 0802.47056); *P. Benevieri* and *M. Furi*, Ann. Sci. Math. Qué. 22, No. 2, 131–148 (1998; Zbl 1058.58502)] that one may consider the orientability of the differential instead.

In contrast to the analytic definitions of orientability devised in the papers mentioned above, here the author follows a more geometric approach: a continuous family of Fredholm operators of index 0 is called orientable if the corresponding naturally defined determinant line bundle is trivial. Under certain connectivity restrictions on the parameter space it is then shown that these notions of orientability coincide if the operator family contains an invertible operator. It should be remarked that this idea also appears in [*A. Floer* and *H. Hofer*, Math. Z. 212, 13–38 (1993; Zbl 0789.58022)].

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MSC:

58B15	Fredholm structures on infinite-dimensional manifolds
47H11	Degree theory for nonlinear operators
58C30	Fixed-point theorems on manifolds

Keywords:

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