Jezierski, Jerzy; Marzantowicz, Wacław

Homotopy methods in topological fixed and periodic points theory. (English) Zbl 1085.55001

Topological Fixed Point Theory and Its Applications 3. Berlin: Springer (ISBN 1-4020-3930-1). xi, 319 p. (2006).

This is an up-to-date exposition of the topological fixed and periodic point theories of selfmaps of finite polyhedra. The phrase "homotopy methods" in its title refers to the fact that the theories presented depend on techniques that come from algebraic topology and thus are homotopy invariant. The book begins with a careful exposition of the fixed point index and Lefschetz number, leading up to the Lefschetz-Hopf Theorem that relates these two fundamental tools of fixed point theory. There is also a discussion of the role of the choice of coefficients in defining the Lefschetz number. Turning to periodic points, the focus is on the sequence of integers obtained by calculating the Lefschetz numbers of the iterates of a selfmap and the several natural conditions under which the unboundedness of that sequence implies that the map has infinitely many periodic points. A careful and detailed exposition of the Nielsen number, that furnishes a lower bound for the number of fixed points of a map, emphasizes the tools that are available to calculate it. The principal geometric result in this subject, the Wecken Theorem that in most circumstances there is a homotopy of the given map to one with exactly the Nielsen number of fixed points, is given a clear and well-organized proof.

The authors then apply Nielsen theory to the investigation of periodic points, presenting a detailed proof of the recent Wecken Periodic Point Theorem of the first author. This chapter also includes useful computational results of Heath and Keppelmann. A positive integer m is a homotopy minimal period of a map f if every map homotopic to f has a periodic point of period m that is not a periodic point of lower period. The theory of homotopy minimal periods, much of it due to these authors, is well described along with some other topics in periodic point theory that make use of the Nielsen number. Thus the contents span a great deal of mathematics, but this reviewer regrets that there is no mention of the use of the Lefschetz number and Wecken's Theorem for the examples of Fadell and Husseini that show that the fixed point property is not a Cartesian product invariant, even for manifolds. Moreover, although there is a brief discussion of the failure of Wecken's theorem for maps of surfaces, that important development merits a more detailed exposition.

There is a carefully-selected bibliography, an index of the authors mentioned in the text as well as a traditional index and also an index of symbols. The earlier chapters contain some exercises. The authors use of English is accurate for the most part but, unfortunately, it often fails to be idiomatic. This failing should not affect the readers' comprehension of the mathematics, but it can be quite distracting. The publisher is at fault for not arranging for the non-technical copyediting that could easily have rectified this problem. This fine book is the first extensive exposition of this sort of topological fixed point theory and related topics since Jiang's lecture notes of 1983 and the fact that it is three times as long is an accurate reflection of the growth of this subject since that time. Two developments after 1983 that have had a great impact and deepened the ties with dynamics are Anosov's theorem concerning Nielsen theory on homogeneous spaces and the Nielsen theory of periodic points, both of which are well represented here (although Anosov's name fails to appear in the index of authors). It nicely complements the recent book of *A. Granas* and *J. Dugundji* [Fixed Point Theory, Springer Monographs in Mathematics. New York (2003; Zbl 1025.47002)] which emphasizes the parts of topological fixed point theory that relate to nonlinear analysis.

Reviewer: Robert F. Brown (Los Angeles)

MSC:

55M20 Fixed points and coincidences in algebraic topology
37C25 Fixed points and periodic points of dynamical systems; fixed-point index theory; local dynamics
55-02 Research exposition (monographs, survey articles) pertaining to algebraic topology

Keywords:

fixed point; periodic point; Lefschetz number; Nielsen number; fixed point index