Chambolle, Antonin

An approximation result for special functions with bounded deformation. (English)

Zbl 1084.49038

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A *special displacement with bounded deformation* is a function $u : \Omega \subset \mathbb{R}^N \to \mathbb{R}^N$ whose symmetrized gradient is a bounded measure which coincides, outside an $(N-1)$-dimensional rectifiable *jump set* J_u, with a summable function $e(u)$. In the paper the author proves that in dimension $N = 2$ if u and $e(u)$ are square integrable and $\mathcal{H}^{N-1}(J_u)$ is finite then u can be approximated with a sequence $\{u_n\}$ of piecewise continuous functions whose jump sets J_{u_n} are relatively closed, with u_n and $e(u_n)$ converging strongly in L^2, respectively, to u and $e(u)$ and the lengths $\mathcal{H}^{N-1}(J_{u_n})$ converging to $\mathcal{H}^{N-1}(J_u)$. As an application it is given an Ambrosio-Tortorelli type approximation of a functional which appears in the theory of brittle fracture in linearized elasticity. In the addendum the results are extended to dimension $N \geq 3$ by a careful application of a discretization technique.

Reviewer: Luigi de Pascale (Pisa)

MSC:

49Q20	Variational problems in a geometric measure-theoretic setting
49J45	Methods involving semicontinuity and convergence; relaxation
74G05	Explicit solutions of equilibrium problems in solid mechanics
74R10	Brittle fracture

Keywords:

functions with bounded deformation; free discontinuity problems; brittle fracture; elliptic approximations

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