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Understanding relationships using copulas. (English) Zbl 1081.62564
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Summary: This article introduces actuaries to the concept of “copulas”, a tool for understanding relationships among multivariate outcomes. A copula is a function that links univariate marginals to their full multivariate distribution. Copulas were introduced in 1959 in the context of probabilistic metric spaces. The literature on the statistical properties and applications of copulas has been developing rapidly in recent years. This article explores some of these practical applications, including estimation of joint life mortality and multidecrement models. In addition, we describe basic properties of copulas, their relationships to measures of dependence, and several families of copulas that have appeared in the literature. An annotated bibliography provides a resource for researchers and practitioners who wish to continue their study of copulas. For those who wish to use copulas for statistical inference, we illustrate statistical inference procedures by using insurance company data on losses and expenses. For these data, we (1) show how to fit copulas and (2) describe their usefulness by pricing a reinsurance contract and estimating expenses for pre-specified losses.

MSC:

- 62P05 Applications of statistics to actuarial sciences and financial mathematics
- 60H05 Stochastic integrals
- 62H10 Multivariate distribution of statistics

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