Rohde, Steffen; Schramm, Oded

Basic properties of SLE. (English) Zbl 1081.60069

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The authors study the so called stochastic Loewner evolution (SLE) which is simply a random growth process defined as follows: let B_t be a Brownian motion on $\mathbb{R}$, started from $B_0 = 0$; for $\kappa \geq 0$ let $\xi(t) = \sqrt{\kappa}B(t)$ and for each $z \in \overline{\mathcal{H}}/0$ (where $\overline{\mathcal{H}}$ is the closed upper half plane) let $g_t(z)$ be the solution of the ordinary (stochastic!) differential equation

$$\partial_t g_t(z) = \frac{2}{g_t(z) - \xi(t)}, \quad g_0(z) = z.$$

The parametrized collection of maps $\{g_t : t \geq 0\}$ called the chordal SLE_κ is the central object of study by the authors. The trace γ of SLE is defined by $\gamma(t) = \lim_{z \to 0} \hat{f}_t(z)$ where $f_t = g_t^{-1}$, $\hat{f}_t(z) = f_t(z + \xi(t))$. The authors build up an elaborate set of analytical tools to establish that the trace is a simple path for $\kappa \in [0, 4]$, a self intersecting path for $\kappa \in (4, 8)$ and for $\kappa > 8$, it is a space filling. The authors also establish that the Hausdorff dimension of SLE_κ trace is almost surely at most $1 + \kappa/8$ and that the expected number of disks of size ε needed to cover it inside a bounded set is at least $\varepsilon^{-(1+\kappa/8)+o(1)}$ for $\kappa \in [0, 8)$ along some sequence $\varepsilon \searrow 0$. The paper concludes with a set of interesting conjectures.

Reviewer: S. K. Srinivasan (Chennai)

MSC:

60K35	Interacting random processes; statistical mechanics type models; percolation theory
60H10	Stochastic ordinary differential equations (aspects of stochastic analysis)
82B41	Random walks, random surfaces, lattice animals, etc. in equilibrium statistical mechanics
60J60	Diffusion processes

Keywords:

stochastic Loewner evolution; Brownian motion; Hausdorff dimension

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