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Effective dynamics of magnetic vortices. (English) Zbl 1081.35102
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Summary: We study solutions of Ginzburg-Landau-type evolution equations (both dissipative and Hamiltonian) with initial data representing collections of widely spaced vortices. We show that for long times, the solutions continue to describe collections of vortices, and we identify (to leading order in the vortex separation) the dynamical system describing the motion of the vortex centers (effective dynamics).

MSC:

35Q55 NLS equations (nonlinear Schrödinger equations)
82D55 Statistical mechanics of superconductors
82C26 Dynamic and nonequilibrium phase transitions (general) in statistical mechanics

Cited in **24** Documents

Keywords:

Ginzburg-Landau functional; vortex; effective dynamics; Higgs model; Hamiltonian; energy conservation; strings; Maxwell-Higgs equations; Abrikosov lattice; macroscopic model of superconductivity

Full Text: [DOI](#) [arXiv](#)

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