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Sharp transition between extinction and propagation of reaction. (English) Zbl 1081.35011

J. Am. Math. Soc. 19, No. 1, 251-263 (2006).

Concerning the following one-dimensional reaction-diffusion equation

$$(*) \qquad u_t = u_{xx} + f(u), \quad x \in \mathbb{R}, t > 0.$$

the author proves two long-time convergence results for certain nonlinearities f and initial values of the special type of characteristic functions on intervals. One of them reads as follows:

Theorem 1. Let $\theta_0 \in [0, 1)$ and $f : [0, 1] \to \mathbb{R}$ be Lipschitz with $f(1) = 0$ and such that $f(\theta) = 0$ for all $\theta \in [0, \theta_0]$, and $f(\theta) > 0$ for all $\theta \in (\theta_0, 1)$. If $\theta_0 > 0$, then assume in addition that f is nondecreasing on $[\theta_0, \theta_0+\delta]$ with some $\delta > 0$. Let $u(x, t)$ be a global solution of $(*)$ with the initial value $u(x, 0) = \chi_{[-L,L]}(x)$. Then there exists a constant $L_0 \geq 0$ with the following properties: (i) if $L < L_0$, then $u(x, t) \to 0$ uniformly on $\mathbb{R}$ as $t \to \infty$. (ii) if $L = L_0$, then $u(x, t) \to \theta_0$ uniformly on compact intervals as $t \to \infty$. (iii) if $L > L_0$, then $u(x, t) \to 1$ uniformly on compact intervals as $t \to \infty$.

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MSC:

35B40 Asymptotic behavior of solutions to PDEs
35K57 Reaction-diffusion equations
35K15 Initial value problems for second-order parabolic equations

Keywords:

reaction-diffusion equations; one space dimension

Full Text: DOI arXiv

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