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From diffusion to anomalous diffusion: a century after Einstein's Brownian motion. (English)

Zbl 1080.82022

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Summary: Einstein's explanation of Brownian motion provided one of the cornerstones which underlie the modern approaches to stochastic processes. His approach is based on a random walk picture and is valid for Markovian processes lacking long-term memory. The coarse-grained behavior of such processes is described by the diffusion equation. However, many natural processes do not possess the Markovian property and exhibit anomalous diffusion. We consider here the case of subdiffusive processes, which correspond to continuous-time random walks in which the waiting time for a step is given by a probability distribution with a diverging mean value. Such a process can be considered as a process subordinated to normal diffusion under operational time which depends on this pathological waiting-time distribution. We derive two different but equivalent forms of kinetic equations, which reduce to known fractional diffusion or Fokker-Planck equations for waiting-time distributions following a power law. For waiting time distributions which are not pure power laws one or the other form of the kinetic equation is advantageous, depending on whether the process slows down or accelerates in the course of time.

MSC:

82C70 Transport processes in time-dependent statistical mechanics

82-02 Research exposition (monographs, survey articles) pertaining to statistical mechanics

Full Text: DOI

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