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Transformation formulas in quantum cohomology. (English) Zbl 1077.14080

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It is known that the problem of determining the conditions on conjugacy classes $\overline{A}_{1,\dots,}\overline{A}_s$ in $\mathrm{SU}(n)$, so that these lift to elements $A_{1,\dots,}A_s \in \mathrm{SU}(n)$ with $A_1A_2\dots A_s = 1$, is controlled by quantum Schubert calculus of Grassmannians. *C. Teleman* and *C. Woodward* [Ann. Inst. Fourier 53, 713–748 (2003; Zbl 1041.14025)] have recently generalized this to an arbitrary simple simply connected compact group K. If G is a complex simple group (whose real points are K), then the role played by the Grassmannians is replaced by the flag varieties G/P for P a maximal parabolic subgroup.

In the case of $\mathrm{SU}(n)$ (and similarly for K), there is a natural "action" of the center of $\mathrm{SU}(n)$ on the representation theory side, namely if $c_1,\dots,c_s$ are central elements with $c_1c_2\dots c_s = 1$, then these act on the set of conjugacy classes $\overline{A}_1,\dots,\overline{A}_s$ in $\mathrm{SU}(n)$, liftable to elements $A_1,\dots,A_s$ with $A_1A_2\dots A_s = 1$, the action being just multiplying $\overline{A}_i$ by c_i. This action is well defined on the level of conjugacy classes because the c_i are central. This suggests a natural transformation property of Gromov-Witten numbers of the Grassmannians under the action of the center.

The first aim of the paper is to prove the transformation formulas geometrically and in complete generality (for any simple simply connected complex Lie group). The second is to show that these formulas determine the quantum Schubert calculus in the case of Grassmannians (Bertram's Schubert calculus). The author also gives a strengthening in the case of Grassmannians of a theorem of *W. Fulton* and *C. Woodward* [J. Algebr. Geom. 13, 641–661 (2004; Zbl 1075.14038)] on the lowest power of q appearing in a (quantum) product of Schubert classes in G/P, where P is a maximal parabolic subgroup. Also many results in this paper are new proofs of older results using methods which seem both natural and elementary.

Reviewer: Ivan V. Arzhantsev (Moskva)

MSC:

14N15 Classical problems, Schubert calculus

14N35 Gromov-Witten invariants, quantum cohomology, Gopakumar-Vafa invariants, Donaldson-Thomas invariants (algebro-geometric aspects)

Keywords:

Grassmann variety; parabolic subgroup

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