Dellar, Paul J.

Incompressible limits of lattice Boltzmann equations using multiple relaxation times. (English) Zbl 1076.76063

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Summary: Lattice Boltzmann equations using multiple relaxation times are intended to be more stable than those using a single relaxation time. The additional relaxation times may be adjusted to suppress non-hydrodynamic modes that do not appear directly in the continuum equations, but may contribute to instabilities on the grid scale. If these relaxation times are fixed in lattice units, as in previous work, solutions computed on a given lattice are found to diverge in the incompressible (small Mach number) limit. This non-existence of an incompressible limit is analysed for an inclined one dimensional jet. An incompressible limit does exist if the non-hydrodynamic relaxation times are not fixed, but scaled by the Mach number in the same way as the hydrodynamic relaxation time that determines the viscosity.

MSC:

76M28	Particle methods and lattice-gas methods
65M99	Numerical methods for partial differential equations, initial value and time-dependent initial-boundary value problems
76P05	Rarefied gas flows, Boltzmann equation in fluid mechanics
82C80	Numerical methods of time-dependent statistical mechanics (MSC2010)

Software:

LAPACK; FFTW

Full Text: DOI

References:

[1] E. Anderson, et al. LAPACK Users' Guide. SIAM, Philadelphia, third ed., 1999. Available from http://www.netlib.org/lapack/lug/lapack_lug.html

[2] Behrend, O.; Harris, R.; Warren, P.B., Hydrodynamic behavior of lattice Boltzmann and lattice bhatnagar – gross – krook models, Phys. rev. E, 50, 4586-4595, (1994), doi:10.1103/PhysRevE.50.4586

[3] Benzi, R.; Succi, S.; Vergassola, M., Turbulence modelling by nonhydrodynamic variables, Europhys. lett., 13, 727-732, (1990)

[4] Benzi, R.; Succi, S.; Vergassola, M., The lattice Boltzmann equation: theory and applications, Phys. rep., 222, 145-197, (1992)

[5] Bhatnagar, P.L.; Gross, E.P.; Krook, M., A model for collision process in gases. I. small amplitude processes in charged and neutral one-component system, Phys. rev., 94, 511-525, (1954), doi:10.1103/PhysRev.94.511 · Zbl 0055.23609

[6] Cercignani, C., The Boltzmann equation and its applications, (1988), Springer-Verlag New York · Zbl 0646.76001

[7] Chapman, S.; Cowling, T.G., The mathematical theory of non-uniform gases, (1991), Cambridge University Press Cambridge · Zbl 0726.76084

[8] Chen, S.; Chen, H.; Martínez, D.O.; Matthaeus, W.H., Lattice Boltzmann model for simulation of magnetohydrodynamics, Phys. rev. lett., 67, 3776-3779, (1991), doi:10.1103/PhysRevLett.67.3776

[9] Chen, S.; Doolen, G.D., Lattice Boltzmann method for fluid flows, Annu. rev. fluid mech., 30, 329-364, (1998) · Zbl 1398.76180

[10] Das, S.P.; Bussemaker, H.J.; Ernst, M.H., Generalized hydrodynamics and dispersion relations in lattice gases, Phys. rev. E, 48, 245-255, (1993), doi:10.1103/PhysRevE.58.245

[11] Dellar, P.J., Bulk and shear viscosities in lattice Boltzmann equations, Phys. rev. E, 64, 031203, (2001), doi:10.1103/PhysRevE.64.031203

[12] Dellar, P.J., Nonhydrodynamic modes and a priori construction of shallow water lattice Boltzmann equations, Phys. rev. E, 65, 036309, (2002), doi:10.1103/PhysRevE.65.036309

[13] d'Humières, D., Generalized lattice-Boltzmann equations, (), 450-458

[14] d'Humières, D.; Ginzburg, I.; Krafczyk, M.; Lallemand, P.; Luo, L.-S., Multiple-relaxation-time lattice Boltzmann models in three dimensions, Phil. trans. R. soc. lond. A, 360, 437-451, (2002), doi:10.1098/rsta.2001.0955 · Zbl 1001.76081

[15] M. Frigo, S.G. Johnson, FFTW: An adaptive software architecture for the FFT, in: Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing, Seattle, 1998, vol. 3, pages 1381-1384, Piscataway, NJ, 1998. IEEE Press. Available from http://www.fftw.org/

[16] Frisch, U.; d'Humières, D.; Hasslacher, B.; Lallemand, P.; Pomeau, Y.; Rivet, J.-P., Lattice gas hydrodynamics in two and three dimensions, Complex sys., 1, 649-707, (1987) · Zbl 0662.76101

[17] Golub, G.H.; Van Loan, C.F., Matrix computations, (1996), Johns Hopkins University Press Baltimore · Zbl 0865.65009

[18] Grad, H., Principles of the kinetic theory of gases, (), 205-294

[19] He, X.; Chen, S.; Doolen, G.D., A novel thermal model of the lattice Boltzmann method in incompressible limit, J. comput. phys., 146, 282-300, (1998), doi:10.1006/jcph.1998.6057 · Zbl 0919.76068

[20] He, X.; Luo, L.-S., Lattice Boltzmann model for the incompressible navier – stokes equation, J. stat. phys., 88, 927-944, (1997) · Zbl 0939.82042

[21] He, X.; Luo, L.-S., Theory of the lattice Boltzmann method: from the Boltzmann equation to the lattice Boltzmann equation, Phys. rev. E, 56, 6811-6817, (1997), doi:10.1103/PhysRevE.56.6811

[22] He, X.; Shan, X.; Doolen, G.D., Discrete Boltzmann equation model for nonideal gases, Phys. rev. E, 57, R13-R16, (1998)

[23] Higuera, F.J.; Jiménez, J., Boltzmann approach to lattice gas simulations, Europhys. lett., 9, 663-668, (1989)

[24] Higuera, F.J.; Succi, S.; Benzi, R., Lattice gas dynamics with enhanced collisions, Europhys. lett., 9, 345-349, (1989)

[25] Lallemand, P.; Luo, L.-S., Theory of the lattice Boltzmann method: dispersion, dissipation, isotropy, Galilean invariance, and stability, Phys. rev. E, 61, 6546-6562, (2000)

[26] Luo, L.-S.; Chen, H.; Chen, S.; Doolen, G.D.; Lee, Y.-C., Generalized hydrodynamic transport in lattice-gas automata, Phys. rev. A, 43, 7097-7100, (1991), doi:10.1103/PhysRevA.43.7097

[27] McNamara, G.R.; Garcia, A.L.; Alder, B.J., Stabilization of thermal lattice Boltzmann models, J. stat. phys., 81, 395-408, (1995) · Zbl 1106.82353

[28] Minion, M.L.; Brown, D.L., Performance of under-resolved two-dimensional incompressible flow simulations, II, J. comput. phys., 138, 734-765, (1997), doi:10.1006/jcph.1997.5843 · Zbl 0914.76063

[29] Qian, Y.H.; d'Humières, D.; Lallemand, P., Lattice BGK models for the navier – stokes equation, Europhys. lett., 17, 479-484, (1992) · Zbl 1116.76419

[30] Skordos, P.A., Initial and boundary conditions for the lattice Boltzmann method, Phys. rev. E, 48, 4823-4842, (1993), doi:10.1103/PhysRevE.48.4823

[31] Sterling, J.D.; Chen, S., Stability analysis of lattice Boltzmann methods, J. comput. phys., 123, 196-206, (1996), doi:10.1006/jcph.1996.0016 · Zbl 0840.76078

[32] Succi, S., The lattice Boltzmann equation: for fluid dynamics and beyond, (2001), Oxford University Press Oxford · Zbl 0990.76001

[33] Succi, S.; Karlin, I.V.; Chen, H., Role of the H theorem in lattice Boltzmann hydrodynamic simulations, Rev. mod. phys., 74, 1203-1220, (2002), doi:10.1103/RevModPhys.74.1203

[34] Uhlenbeck, G.E.; Ford, G.W., Lectures in applied mathematics, Lectures in statistical mechanics, vol. 1, (1963), American Mathematical Society Providence · Zbl 0111.43802

[35] Zou, Q.S.; Hou, S.L.; Chen, S.Y.; Doolen, G.D., An improved incompressible lattice Boltzmann model for time-independent flows, J. stat. phys., 81, 35-48, (1995) · Zbl 1106.82366