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Recovering an algebraic curve using its projections from different points. Applications to static and dynamic computational vision. (English) Zbl 1070.14051

Summary: We study some geometric configurations related to projections of an irreducible algebraic curve embedded in \( \mathbb{CP}^3 \) onto embedded projective planes. These configurations are motivated by applications to static and dynamic computational vision. More precisely, we study how an irreducible closed algebraic curve \( X \) embedded in \( \mathbb{CP}^3 \), of degree \( d \) and genus \( g \), can be recovered using its projections from points onto embedded projective planes. The embeddings are unknown. The only input is the defining equation of each projected curve. We show how both the embeddings and the curve in \( \mathbb{CP}^3 \) can be recovered modulo some action of the group of projective transformations of \( \mathbb{CP}^3 \). In particular in the case of two projections, we show how in a generic situation, a characteristic matrix of the pair of embeddings can be recovered. In the process we address dimensional issues and as a result find the minimal number of irreducible algebraic curves required to compute this characteristic matrix up to a finite-fold ambiguity, as a function of their degrees and genus. Then we use this matrix to recover the class of the couple of maps and as a consequence to recover the curve. In a generic situation, two projections define a curve with two irreducible components. One component has degree \( d(d - 1) \) and the other has degree \( d \), being the original curve. Then we consider another problem. \( N \) projections, with known projection operators and \( N \gg 1 \), are considered as an input and we want to recover the curve. The recovery can be done by linear computations in the dual space and in the Grassmannian of lines in \( \mathbb{CP}^3 \). Those computations are respectively based on the dual variety and on the variety of intersecting lines. In both cases a simple lower bound for the number of necessary projections is given as a function of the degree and the genus. A closely related question is also considered. Each point of a finite closed subset of an irreducible algebraic curve is projected onto a plane from a point. For each point the projection center is different. The projection operators are known. We show when and how the recovery of the algebraic curve is possible, in terms of the degree of the curve, and of the degree of the curve of minimal degree generated by the projection centers.

MSC:
14N05 Projective techniques in algebraic geometry
14H50 Plane and space curves
68T45 Machine vision and scene understanding
14Q05 Computational aspects of algebraic curves

Software:
FGb; SINGULAR

Full Text: Link arXiv

References:


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